Asset price dynamics with heterogeneous beliefs and local network interactions

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CompEc

In this paper authors investigate the effects of network topologies on asset price dynamics. They introduce network communications into a simple asset pricing model with heterogeneous beliefs. The agents may switch between several belief types according to their performance. The performance information is available to the agents only locally through their own experience and the experience of other agents directly connected to them.

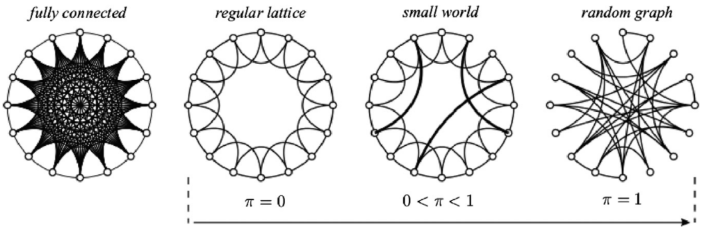

Fig. 1. Network topologies (adapted from Watts and Strogatz, 1998). π indicates a link rewiring probability.

K=degree and N=the total number of nodes. In the fully connected network, all nodes are linked to all other nodes. In the regular lattice, each node is linked to a fixed number of neighboring nodes, and hence, all nodes have the same degree, which we denote by K. In order to form a small world network, a link is rewired to a different randomly chosen node on the lattice (avoiding self- and double-connections) with a given rewiring probability $\pi \in (0, 1)$. Such rewiring of the nodes continues until all the links are processed. In the limit when $\pi = 1$ the network becomes similar to the random graph of Erdos and Rényi (1959), with N nodes and the probability of the link between any two nodes equals to $K/N - 1$.

BH1998, 'Heterogeneous beliefs and routes to chaos in a simple asset pricing model', JEDC Vol.22, Issue 8-9

- It is a financial market application of BH1997
- There is bounded rationality
- Agents select their strategy rule based upon recent relative performance
- It is a stylized model, to some extent analytically tractable
- Really good: it is formulated in terms of deviation from a RE benchmark

- Agents can invest in risk free or in a risky asset
- The risk free is perfectly elastically supplied and pays an interest r. While the risky asset pays and uncertain dividend;
- Agents are myopic mean-variance maximizers so the demand per trader of type h for the risky asset is:

$$z_{ht} = \frac{E_{ht}[p_{t+1} + y_{t+1} - (1+r)p_t]}{a\sigma^2}$$

  where E_{ht} are belief of trader type h, and y_{t+1} is the dividend, and a is the risk aversion parameter.
- z^s is the constant supply and n_{ht}

- therefore, the equilibrium between demand and supply is

$$\sum_{h=1}^{H} n_{ht} \frac{E_{ht}[p_{t+1} + y_{t+1} - (1+r)p_t]}{a\sigma^2} = z^s$$

- BH98 assumes that $z^s = 0$, therefore the Walrasian market clearing price satisfies:

$$(1+r)p_t = \sum_{h=1}^{H} n_{ht} E_{ht}[p_{t+1} + y_{t+1}]$$

- in an homogeneous world:

$$p_t^* = \sum_{k=1}^{\infty} \frac{E_t[y_{t+k}]}{(1+r)^k}$$

assuming that the dividend is equal for each period

$$p^* = \sum_{k=1}^{\infty} \frac{\bar{y}}{(1+r)^k} = \frac{\bar{y}}{r}$$

Heterogenous Beliefs:

- in a heterogeneous world traders can deviate from the fundamental price:

$$x_t = p_t - p_t^*$$

- and beliefs are:

$$E_{ht}[p_{t+1}] = E_t[p_{t+1}^*] + f_h(x_{t-1}, \ldots .x_{t-L}) = E_t[p_{t+1}^*] + f_{ht}$$

  where f_h represents a strategy rule.

- therefore

$$(1+r)x_t = \sum_{h=1}^{H} n_{ht} f_{ht}$$

- Evolutionary selection strategies follow a discrete choice approach

$$n_{ht} = \frac{exp(\beta U_{h,t-1})}{\sum_{h=1}^{H} exp(\beta U_{h,t-1})}$$

- in BH the evolutionary fitness is given by the relized profits

$$U_{ht} = (p_t + y_t - Rp_{t-1}) \frac{E_{ht}[p_t + y_t - Rp_{t-1}]}{a\sigma^2} + \omega U_{h,t-1}$$

where ω is a memory parameter. Let's assume that $\omega = 0$.

- therefore:

$$U_{ht} = (x_t - Rx_{t-1}) \frac{f_{h,t-1} - Rx_{t-1}}{a\sigma^2}$$

Let's assume that:

$$f_{ht} = g_h x_{t-1} + b_n$$

where g_h is a trend parameter and b_n a bias parameter. Therefore

1. fundamentalist: $g_h = b_n = 0$
2. trend-followers-chartists: $b_n = 0$
3. biased belief: $g_h = 0$

$$E^h_{t-1}[p_{t+1} + y_{t+1}] = b^h + p^* + g^h(p_{t-1} - p^*)$$

b^h is a constant bias and g^h is an extrapolation parameter. They assume $b^h = 0$. For fundamentalist $g^h = 0$, while chartists expect persistent deviations from the fundamental value and use a positive extrapolation parameter, $g > 0$.

- the agents are located on the nodes of a network and can observe the performance measure of the predictor types employed only by those agents who reside on the nodes directly connected with them.
- Hence, they cannot observe the performance of the types adopted by agents located two or more links away.
- Therefore, contrary to the BH model, the performance of every type is available to all the agents. Only for local information exchange in the market.

- if an agent is directly connected only to the agents of the same type, they are not able to switch as there is no information about the performance of the alternative type(s).
- If an agent has at least one neighbor of a different type, they are able to compare the utility from their own type with the utility from the alternative observed type(s) and make a choice.
- Note that under local information exchange, the fractions of the belief types do not follow the discrete choice because some agents are not able to switch.

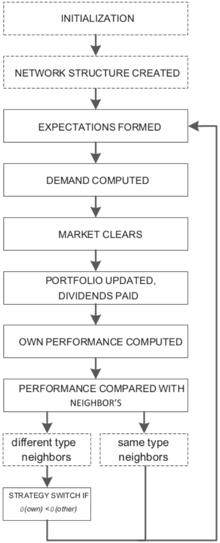

Fig. 3. Temporal flow.

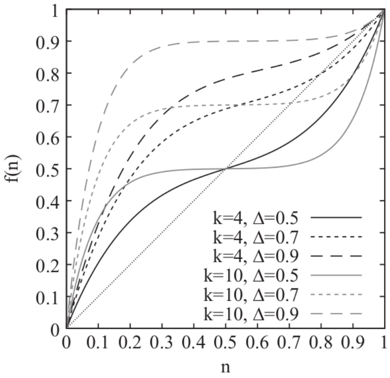

Fig. 4. Dependence of map $f(n) = n^{k+1} + (1-n^{k+1}-(1-n)^{k+1})\Delta$, on k and Δ.

where Δ denotes the steady state fraction of chartists in the original BH model. An important difference from the BH model is that in this model in addition to the 'interior' fundamental steady state, they observe two 'corner' fundamental states which are E_0 with all agents being fundamentalists and E_1 with all agents being chartists.

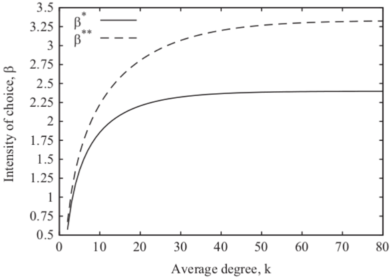

Fig. 5. Bifurcation values of β for given k, $g = 1.2$, $R = 1.1$, $c = 1$.

The bifurcation value, β^*, is lower in this model with the random graph than in the original BH model. At $\beta = \beta^{**}$ nn the (secondary) Neimark-Sacker bifurcation takes place and the non-fundamental steady states loose stability. The bifurcation values of β are increasing in k and converge to the original BH.

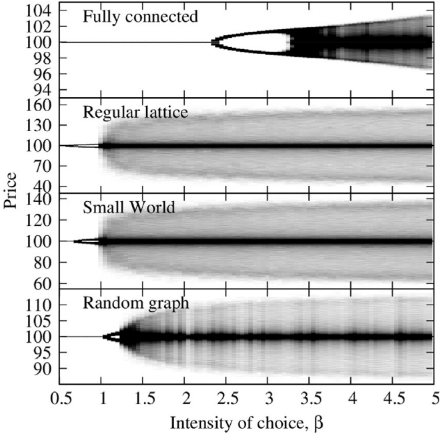

Fig. 6. Bifurcation diagrams for various network topologies.

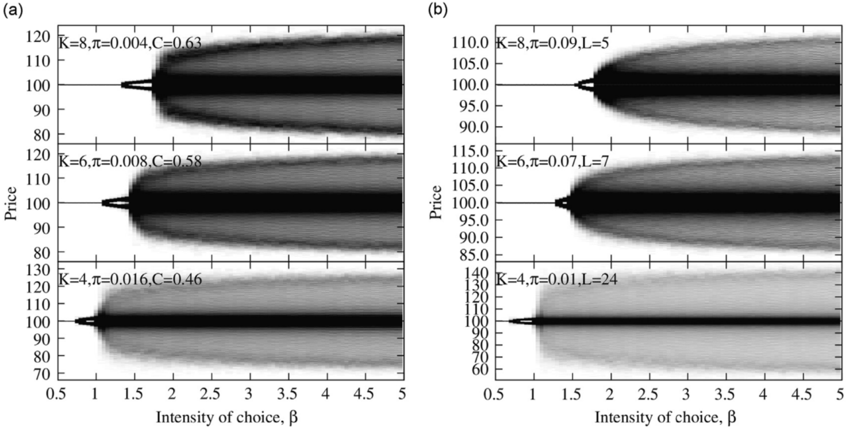

Fig. 7. Bifurcation diagrams for varying K, π, L and C. (a) Fixed $L = 17$. (b) Fixed $C = 0.48$.

The informational efficiency is closely related to the speed of information transmission and can be measured by comparing the volatility of the observed price with the volatility of the fundamental dividend process as suggested by Shiller (1981).

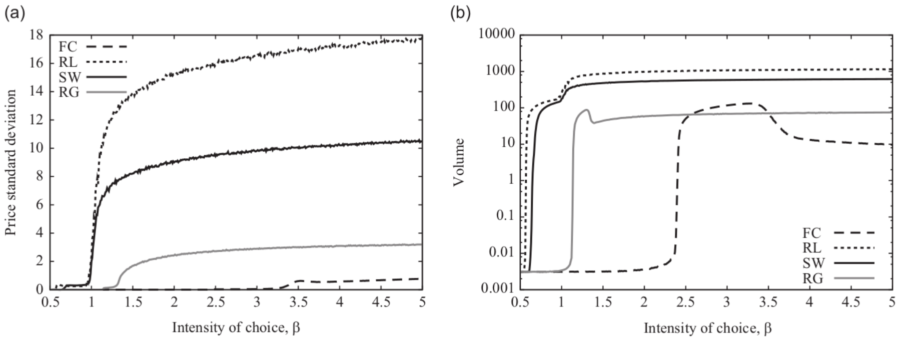

Fig. 10. Measures of market information inefficiency. (a) Standard deviation of price. (b) Average traded volume.

They observe that the random graph and the fully connected network exhibit the most informational efficient outcomes for any values of β which is consistent with the highest speed of information transmission in these two networks. The regular lattice exhibits the least informationally efficient outcome.

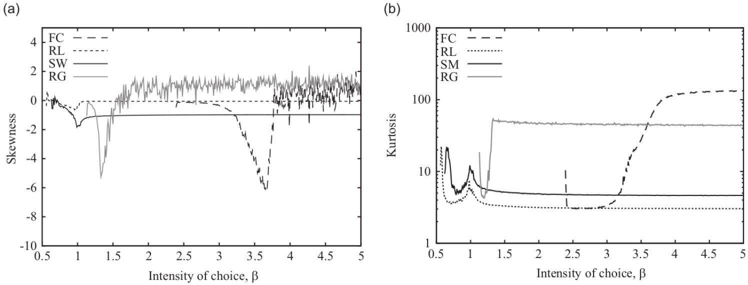

Fig. 11. (a) Skewness and (b) Kurtosis of returns.

The skweness measures the asymmetry of the distribution. It is close to zero for all the networks for all post-bifurcation values of β. The returns generated by the model with the small world network are slightly negatively skewed. The small world network return distribution exhibits the kurtosis value around 8, which is relatively close to the one observed for the returns on the financial markets.

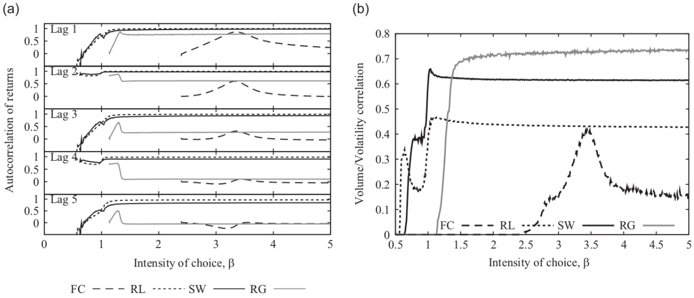

Fig. 12. Properties of returns (a) Autocorrelations of returns and (b) Volume/volatility correlations.

The graph on the left depicts the autocorrelation of returns for the first five lags as a function of the intensity of choice. Usually empirical stock return series exhibit small or no autocorrelation. The regular lattice and the small world network produce high autocorrelations at all lags. Although the random graph and the fully connected network display large autocorrelations at the first two lags, they converge to zero autocorrelation values at lags three to five. The significant positive autocorrelations are resulting predominantly from the persistence of the chartist type.

Table 1

Characteristics depending on the network in increasing order left to right. The characteristics depending on values of β are compared at fixed β: $\beta = 4$ for the two-type model and $\beta = 85$ for the four-type model. *Note*: $A \approx B$ indicates that there is no clear ranking between A and B, *neg* stands for negative values.

Characteristic	Two-type model	Four-type model
Latency in information transmission	FC RG SW RL	FC RG SW RL
$1/\beta$ of the primary bifurcation	FC RG SW RL	FC RG SW $\approx$ RL
Length of instability interval	FC RG SW RL	FC RG SW $\approx$ RL
Amplitude of price fluctuation	FC RG SW RL	RL SW FC RG
Std. deviation of price	FC RG SW RL	RL SW FC RG
Average trading volume	FC RG SW RL	RL SW FC RG
Skewness of returns	SW RL FC RG	RL SW FC RG
Kurtosis of returns	RL SW RG FC	SW RL FC RG
Autocorrelations of returns	FC RG SW RL	inconclusive
Volume/volatility correlations	FC RL SW RG	FC (*neg*) SM RL RG
Autocorrelations of squared returns	FC RG SW RL	FC RG RL SW

- the BH model (1998) is expanded by introducing local information exchange via communication networks.
- Authors study how different network structures affect asset price dynamics.
- They derive a low-dimensional system to represent dynamics in the two-type model with a random graph and prove some stability results for this case.
- Other network structures are investigated by simulations. They observe that the stability regions with respect to the intensity of choice parameter depend on the parameters of the communication network.
- In the two-type model, the latency in the information transmission, which is the highest for the regular lattice and the small world networks, creates greater information inefficiencies and induces greater instabilities and higher deviations in the price dynamics.
- (Non showed) In the four-type network model, the latency in information transmission causes qualitatively different results.