Endogenous Cycles in Heterogeneous Agent Models. A State-Space Approach

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Intro

- HAMs **theoretical** literature is now mature;
- A growing (almost mature) **empirical** literature, even if we cannot observe any consensus on the estimation methodology.
- There are also experiments (See Ch. 8. Hommes' book)

Franke and Westerhoff (2017) note two approaches: direct and indirect.

1. The first method employs surveys to measure the sentiments of a specific group of the population;
2. The second considers a model as a whole and strives to estimate its parameters straightaway from the model or on simulating artificial data.

Intro Cont.d

Empirical surveys: Lux and Zwinkels (2018); Ter Ellen and Verschoor (2018)

- Maximum Likelihood Etimation: Alfarano et al. (2006), Chiarella et al. (2014), Ter Ellen et al. (2020), Lux (2021)
- Nonlinear Least Square: Boswijk et al. (2007) **in Chapter 7 of Hommes' book**, Lof (2012), Hommes and in't Veld (2017)
- Method of Simulated Moment: Franke and Westerhoff (2012), Schmitt (2021)
- Markov RS: Chiarella et al. (2012)
- Non Parametric Simulated MLE: Kukacka and Barunik (2017)
- Vector Error Correction Model: Frijns and Zwinkels (2018)
- **State Space Model**: Thomas Lux (2018, 2021), Gusella and Stockhammer (2021), Gusella (2022)

Motivations

1. State-space model allows to treat the beliefs as unobservable variables:

$$P_t = P_t^f + \sum B_t^i \left(P_{t-1}^f, P_{t-1} \right)$$

2. Empirical works have focused more on finding heterogeneity in data, while few works, especially for stock market prices, have devoted attention to the possibility of extrapolating forecasting power through the models used. Manzan and Westerhoof (2007), De Jong et al. (2010) and Jongen et al. (2012) for exchange rates market, Chiarella et al. (2012), Lof (2012), Recchioni et al. (2015) for the stock market

3. State-space models have not been used to study the forecasting performance of (this family of) heterogeneous agents models.

4. In HAMs literature for stock prices, the fundamental value follows a random walk, as in Franke (2008), or it is constructed from the Gordon growth model as in Chiarella et al. (2012). However, the forecasting power of the two specifications has never been compared.

Research question and method

The research question:

Is there empirical evidence of financial cycles in asset prices when the price dynamics is driven by the interaction between fundamentalists and chartists?

The research method:

1. The state-space model
2. The Kalman filter algorithm for the estimation procedure + Monte Carlo simulations
3. State-space rolling-window in-sample
4. State-space rolling-window forecasting analysis (out of sample)
5. S&P500 at different frequency: quarterly, monthly and daily (1990 - 2020)

Differences with related papers

1. Thomas Lux (2018)
   1.1 Bayesian approach vs frequentist approach
   1.2 Single parameters estimation vs cycles estimation
   1.3 Time varying unobserved quotas vs unobserved belief functions with fixed quotas
2. Gusella and Stockhammer (2021)
   Gusella (2022)
   2.1 Different unobserved components
   2.2 Different expectations
   2.3 The role of the fundamental value
   2.4 Time frequency analysis

Methodology

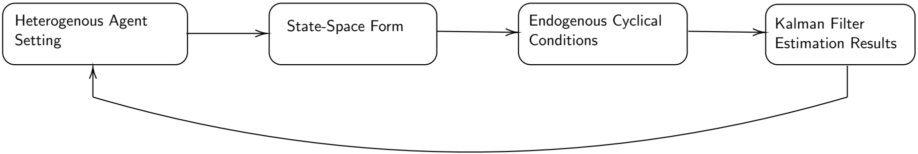


Figure: From the heterogeneous agent setting to the state-space model, and back.

Methodology: HAMs setting

The financial operators:

$$W_{t+1}^{i} = (1+r)W_{t}^{i} + \left[p_{t+1} + y_{t+1} - (1+r)p_{t}\right] Z_{t}^{i} \tag{1}$$

The asset price:

$$p_t = \delta E_t^f \left(p_{t+1}\right) + \left(1-\delta\right) E_t^s \left(p_{t+1}\right) \tag{2}$$

Fundamentalists:

$$E_t^f \left(p_{t+1}\right) = p_t^f \tag{3}$$

Chartists/Speculators:

$$E_t^s \left(p_{t+1}\right) = p_t^f + \beta \left(p_{t-1} - p_{t-2}\right), \qquad with \quad \beta > 0 \tag{4}$$

Methodology: HAMs setting

$$p_t = p_t^f + (1 - \delta)\, B_t^s \tag{5}$$

where:

$$B_t^s = \beta\,(p_{t-1} - p_{t-2}) \tag{6}$$

and from a Gordon model:

$$P_t{}^f = u_t = d_t \frac{(1+g)}{\tilde{y}} \tag{7}$$

From HAMs setting to State Space Framework

$$B_t^s = \beta\left(p_{t-1}^f + (1-\delta)\, B_{t-1}^s - p_{t-2}^f - (1-\delta)\, B_{t-2}^s\right).$$

In line with Franke (2008), the fundamental value is governed by an exogenous univariate stochastic process, such that differentiating between p_{t-1}^f and p_{t-2}^f does not matter (we test this hypothesis with a Dickey-Fuller test). We thus have:

$$B_t^s = \beta\,(1-\delta)\, B_{t-1}^s - \beta\,(1-\delta)\, B_{t-2}^s.$$

An essential feature of any state space model is such that the state equation must be a first-order stochastic difference equation (Commandeur and Koopman, 2007; Enders, 2016)

Methodology: State Space Form

The observation equation:

$$p_t = p_t^f + \left[\begin{array}{cc} (1-\delta) & 0 \end{array} \right] \left[\begin{array}{c} B_t^s \\ B_{t-2}^s \end{array} \right] \tag{8}$$

The transition equation:

$$\left[\begin{array}{c} B_t^s \\ B_{t-1}^s \end{array} \right] = \left(\begin{array}{cc} \beta(1-\delta) & -\beta(1-\delta) \\ 1 & 0 \end{array} \right) \left[\begin{array}{c} B_{t-1}^s \\ B_{t-2}^s \end{array} \right] + \left[\begin{array}{c} \varepsilon_t \\ 0 \end{array} \right] \tag{9}$$

For simplicity on notations, we set

$$\begin{cases} \beta(1-\delta) = a_{11} \\ -\beta(1-\delta) = a_{12} \end{cases} \tag{10}$$

Methodology: State Space Form

Summing up, the state-space model can be rewritten in compact form as:

$$p_t = p_t^f + HB_t,$$

where p_t **is the observable asset price**, and

$$H = \left[\begin{array}{cc} (1-\delta) & 0 \end{array} \right], \quad \textit{is} \quad \textbf{the measurement matrix}$$

$$B_t = AB_{t-1} + \varphi_t \qquad \varphi_t \sim \mathrm{N}(0, Q),$$

$$B_t = \left[\begin{array}{c} B^s_{t-1} \\ B^s_{t-2} \end{array} \right], \quad \textit{is} \quad \textbf{the state vector}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ 1 & 0 \end{pmatrix}, \quad \textit{is} \quad \textbf{the transition matrix}$$

and φ_t is the vector containing the state disturbance of the unobserved component, normally distributed with mean zero and variances collected in the diagonal matrix Q.

Methodology: The cyclical conditions

$$\beta(1-\delta)[\beta(1-\delta)-4]<0 \tag{11}$$

From the previous equation, we observe that the necessary and sufficient condition to obtain complex eigenvalues reads as follow:

$$0<(1-\delta)\beta<4$$

If it holds, the two associated complex eigenvalues assume the following form:

$$\lambda_{1,2}=\frac{Tr(A)}{2}\pm i\frac{\sqrt{-\Delta}}{2}=a\pm ib, \quad with \quad A=\begin{pmatrix}\beta(1-\delta) & -\beta(1-\delta)\\ 1 & 0\end{pmatrix}.$$

If $\sqrt{\beta(1-\delta)}=1$, we obtain constant-amplitude endogenous fluctuations. With the modulus of eigenvalue lower than one ($\sqrt{\beta(1-\delta)}<1$), we observe damped endogenous fluctuations, while explosive endogenous fluctuations if $\sqrt{\beta(1-\delta)}>1$.

Data

- S&P 500 between 1990-2020 (in sample)
- We run the model between 1990-2017
- We forecast 2018-2020 using a rolling window
- agents forecast at the different horizons (quarterly, monthly, daily)

Table: Estimation Results [S&P500 Quarterly Data]

	Cyclical Parameters			
	a_{11}	a_{12}	a_{21}	a_{22}
Values	0.42* (0.26082)	-0.42* (0.26082)	1	0

	Percentage and Reaction Coefficients	
	$1-\delta$	β
Values	0.35*** (0.01122)	1.2

	State Disturbance
	σ_{ε}
Values	0.89080*** (0.02581)

	Cyclical Conditions
$\beta(1-\delta)[\beta(1-\delta)-4]<0$	*Fulfilled*
$\left(a^2+b^2\right)^{1/2}$	*Damped*

	Info Model
Eigenvalues	$-0.2102 \pm 0.6134i$
Modulus	0.65
Log-likelihood	-31.7849
Akaike Info Criterion	71.5697
Bayesian Info Criterion	82.8184

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Table: Estimation Results [S&P500 Monthly Data]

	Cyclical Parameters			
	a_{11}	a_{12}	a_{21}	a_{22}
Values	0.53***	-0.53***	1	0
	(0.23568)	(0.23568)		
	Percentage and Reaction Coefficients			
	$1-\delta$	β		
Values	0.63***	0.85		
	(0.01004)			
	State Disturbance			
	σ_ε			
Values	0.48935***			
	(0.00783)			
	Cyclical Conditions			
$\beta(1-\delta)[\beta(1-\delta)-4]<0$	*Fulfilled*			
$(a^2+b^2)^{1/2}$	*Damped*			
	Info Model			
Eigenvalues	$-0.2688 \pm 0.6822i$			
Modulus	0.73			
Log-likelihood	-89.2338			
Akaike Info Criterion	186.468			
Bayesian Info Criterion	202.143			

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Table: Estimation Results [S&P500 Daily Data]

	Cyclical Parameters			
	a_{11}	a_{12}	a_{21}	a_{22}
Values	0.43* (0.24365)	-0.43* (0.24365)	1	0
	Percentage and Reaction Coefficient			
	$1-\delta$	β		
Values	0.71*** (0.00252)	0.60		
	State Disturbance			
	σ_ε			
Values	0.00151*** (0.00783)			
	Cyclical Conditions			
$\beta(1-\delta)[\beta(1-\delta)-4]<0$	*Fulfilled*			
$(a^2+b^2)^{1/2}$	*Damped*			
	Info Model			
Eigenvalues	$-0.2165 \pm 0.6214i$			
Modulus	0.66			
Log-likelihood	-1890.21			
Akaike Info Criterion	3788.42			
Bayesian Info Criterion	3816.28			

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Comparison

1. Our results support endogenous cycles for quarterly and daily data at ten percent level and at one percent level for monthly data
2. The modulus of complex eigenvalues near one to generate persistent oscillations (at least for mothly data)
3. The percentage of chartists tends to increase in correspondence of an increase of frequency time
4. The intensity of reaction parameters decreases passing from quarterly to monthly and daily data

Table: Comparison between the different frequencies

		Behavioral HAM		
		Quarterly	*Monthly*	*Daily*
Pct. of fundamentalists	δ	0.65	0.37	0.29
Pct. of speculators	$1-\delta$	0.35	0.63	0.71
Spec. reaction coeff.	β	1.2	0.88	0.60
Akaike info criterion	*AIC*	71.5697	186.468	3788.42
Bayesian info criterion	*BIC*	82.8184	202.143	3816.28
Modulus	ρ	0.65	0.73	0.66
Cycles		*yes*	*yes*	*yes*

State-Space Rolling Window Chartists weight

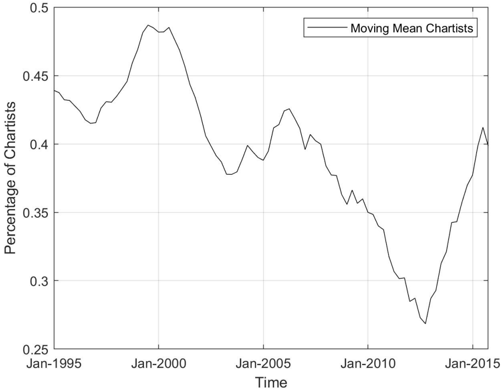


Figure: The line represents the dynamics of the moving mean of the parameter $(1-\delta)$ over a length 5 sliding window (Quarterly Data).

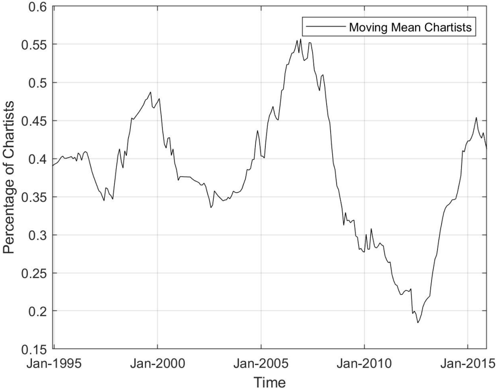


Figure: The line represents the dynamics of the moving mean of the parameter $(1-\delta)$ over a length 13 sliding window (Monthly Data).

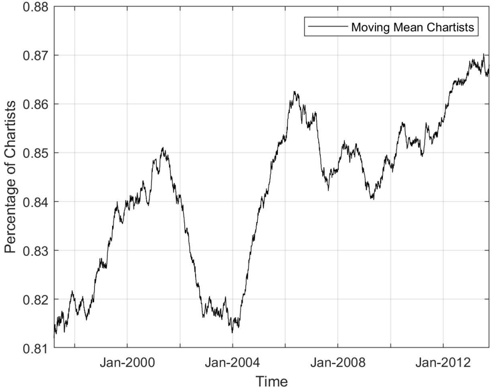
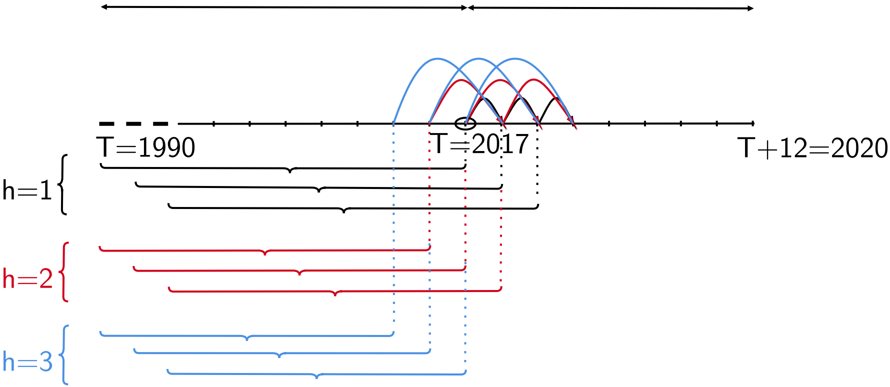


Figure: The line represents the dynamics of the moving mean of the parameter $(1 - \delta)$ over a length 365 sliding window (Daily Data).

From In-sample to Forecasting



Mincer-Zarnowitz regression test

We run

$$p_{n+h} = b_0 + b_1 p_{n+h|n} + \epsilon_{n+h|n}$$

In order to have:

$$\epsilon_{n+h|n} = p_{n+h} - p_{n+h|n}$$

this two conditions should hold:

$$b_0 = 0, \quad b_1 = 1$$

The regression of the actual value on the ex-ante forecast should have a zero intercept and a coefficient of 1.

Table: Mincer-Zarnowitz regression test

			Quarterly Frequency		
h	1 q. h.	3 q. h.	6 q. h.	9 q. h.	12 q. h.
	1.0248***	1.0303***	1.0242***	1.0248***	1.0393***
			Monthly Frequency		
h	1 m. h.	4 m. h.	8 m. h.	12 m. h.	16 m. h.
	1.0249***	1.0224***	0.9879***	1.0171***	1.0264***
h	20 m. h.	24 m. h.	28 m. h.	32 m. h.	36 m. h.
	1.0256***	1.0251***	1.0252***	1.0253***	1.0393***
			Daily Frequency		
h	1 d. h.	365 d. h.	547 d. h.	730 d. h.	1095 d. h.
	1.0026***	1.0025***	1.0025***	1.0026***	1.0393***

Notes: *h* is the forecasting horizon.

q. h., m. h., d. h. stand for quarterly, monthly and daily horizon respectively.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Table: Out-of-sample forecast results (quarterly frequency)

h	1 q. h.	2 q. h.	3 q. h.	4 q. h.	5 q. h.	6 q. h.
RMSE	1.9254***	2.5118***	2.6063***	1.4911***	1.4437***	1.2067**
	(-3.30)	(-6.37)	(-5.18)	(-3.42)	(-3.81)	(-2.19)
MAE	2.2138***	2.9914***	2.8445***	1.5800***	1.5187***	1.1991
	(-2.98)	(-6.23)	(-5.13)	(-3.00)	(-4.18)	(-1.62)
h	7 q. h.	8 q. h.	9 q. h.	10 q. h.	11 q. h.	12 q. h.
RMSE	1.1167	1.0564	0.8666*	0.8005***	0.7566***	0.7045***
	(-1.60)	(-0.62)	(-1.92)	(-3.25)	(-3.96)	(-5.10)
MAE	1.1667*	1.0678	0.8590**	0.7916***	0.7377***	0.6858***
	(-1.69)	(-0.73)	(-2.22)	(-3.48)	(-4.78)	(-7.47)

Notes: *h* is the forecasting horizon.

q. h. stands for quarterly horizon.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

The columns represent the ratio of root mean squared error (RMSE) and the ratio of the mean absolute forecast error (MAE) of the Behavioral model to that of the RW. A number greater then 1 represents a better performance of the RW hypothesis. *h* represents the forecast month horizons and the Diebold-Mariano t-statistics is in parentheses. The two forecasts have the same accuracy under the null hypothesis

The RW is strongly linked to the short-trend of the current value of the S&P500; therefore, it fails to capture the long-term changes also because it does not carry relevant information. On the other hand, the behavioral model + considering the dividends distributed, captures the profit prospects of the medium-long term and, consequently, allows better predictive performance.

Table: Out-of-sample forecast results (monthly frequency)

h	2 m. h.	4 m. h.	6 m. h.	8 m. h.	10 m. h.	12 m. h.
RMSE	3.9107***	2.0883***	2.1061***	2.0498***	2.0350***	1.8260***
	(-9.77)	(-5.86)	(-8.51)	(-9.17)	(-8.34)	(-7.46)
MAE	4.7643***	2.4895***	2.5597***	2.6038***	2.3257***	1.9929***
	(-14.0)	(-7.72)	(-11.1)	(-10.6)	(-11.9)	(-10.2)
h	14 m. h.	16 m. h.	18 m. h.	20 m. h.	22 m. h.	24 m. h.
RMSE	1.6305***	1.8985***	1.2665***	1.1420***	1.0528	0.9656
	(-7.54)	(-6.10)	(-4.64)	(-2.65)	(-1.14)	(-0.66)
MAE	1.7581***	1.4924***	1.3100***	1.1897***	1.0815*	0.9835
	(-9.85)	(-7.37)	(-5.00)	(-3.42)	(-1.68)	(-0.32)
h	26 m. h.	28 m. h.	30 m. h.	32 m. h.	34 m. h.	36 m. h.
RMSE	0.9157*	0.8497***	0.7972***	0.7609***	0.7311***	0.5908***
	(-1.71)	(-2.96)	(-4.61)	(-6.00)	(-6.95)	(-8.82)
MAE	0.9054**	0.8454***	0.7861***	0.7427***	0.7069***	0.6668***
	(-1.96)	(-3.33)	(-5.31)	(-6.82)	(-8.03)	(-10.8)

Notes: *h* is the forecasting horizon.
m. h. stands for monthly horizon.
*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

The columns represent the ratio of root mean squared error (RMSE) and the ratio of the mean absolute forecast error (MAE) of the Behavioral model to that of the RW. A number greater then 1 represents a better performance of the RW hypothesis. *h* represents the forecast month horizons and the Diebold-Mariano t-statistics is in parentheses. The two forecasts have the same accuracy under the null hypothesis

The RW is strongly linked to the short-trend of the current value of the S&P500; therefore, it fails to capture the long-term changes also because it does not carry relevant information. On the other hand, the behavioral model + considering the dividends distributed, captures the profit prospects of the medium-long term and, consequently, allows better predictive performance.

Conclusions

1. We go beyond the existing empirical literature on HAMs considering the role of endogenous cycles, and showing the importance of the state space models for HAMs

2. This work can support the increasing behavioral theoretical works where the heuristic decisions of agents are considered as the first source of fluctuations

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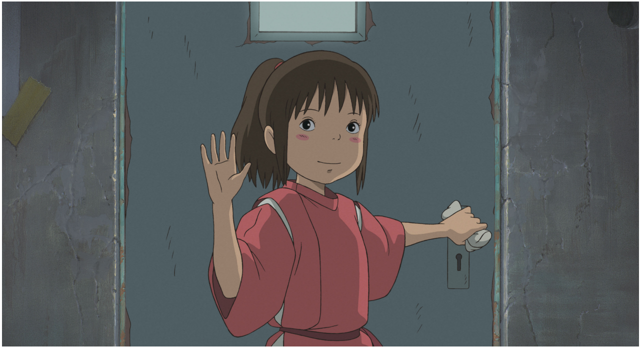

giorgio.ricchiuti@unifi.it (frame de *Spirited Away*, 2004)

Table: Monte Carlo Simulation Results [S&P500 Quarterly Data]

	Cyclical Parameters			
	a_{11}	a_{12}	a_{21}	a_{22}
Values	0.4128 [0.2830 - 0.5430]	-0.4128 [0.2830 - 0.5430]	1	0
	Percentage and Reaction Coefficients			
	$1-\delta$	β		
Values	0.41 [0.32 - 0.49]	1		
	State Disturbance			
	σ_ε			
Values	0.7609 [0.6019 - 0.9199]			
	Cyclical Conditions			
$\beta(1-\delta)[\beta(1-\delta)-4]<0$	*Fulfilled*			
$(a^2+b^2)^{1/2}$	*Damped*			
	Info Model			
Eigenvalues	$0.2064 \pm 0.6084i$			
Modulus	0.64			

Notes: Confidence interval in squared brackets.

Table: Monte Carlo Simulation Results [S&P500 Monthly Data]

	Cyclical Parameters			
	a_{11}	a_{12}	a_{21}	a_{22}
Values	0.5350	-0.5350	1	0
	[0.4639 - 0.6061]	[0.4639 - 0.6061]		
	Percentage and Reaction Coefficients			
	$1-\gamma$	β		
Values	0.51	0.88		
	[0.1873 - 0.8395]			
	State Disturbance			
	σ_{ε}			
Values	0.6832			
	[0.1120 - 1.2544]			
	Cyclical Conditions			
$\beta(1-\delta)[\beta(1-\delta)-4]<0$	*Fulfilled*			
$(a^2+b^2)^{1/2}$	*Damped*			
	Info Model			
Eigenvalues	$-0.2675 \pm 0.6808\mathrm{i}$			
Modulus	0.7315			

Notes: Confidence interval in squared brackets.

Table: Monte Carlo Simulation Results [S&P500 Daily Data]

	Cyclical Parameters			
	a_{11}	a_{12}	a_{21}	a_{22}
Values	0.4227 [0.4064 - 0.4390]	-0.4227 [0.4064 - 0.4390]	1	0
	Percentage and Reaction Coefficients			
	$1-\gamma$	β		
Values	0.76 [0.5635 - 0.9565]	0.55		
	State Disturbance			
	σ_{ε}			
Values	0.4116 [0.2961 - 0.5271]			
	Cyclical Conditions			
$\beta(1-\delta)[\beta(1-\delta)-4]<0$		*Fulfilled*		
$(a^2+b^2)^{1/2}$		*Damped*		
	Info Model			
Eigenvalues	$-0.2114 \pm 0.6148i$			
Modulus	0.6501			

Notes: Confidence interval in squared brackets.