Scuola di Economia e Management — Università di Firenze — a.a. 2022-23

Computational Economics (G.Ricchiuti, Tutor: F. Campigli)

Mock Exam 1

Lets' assume that the following system represents the dynamics of two (endogenous) variables, X and Y:

$$X_{t+1} = 1 - \gamma X_t^2 + Y_t \quad (1)$$
$$Y_{t+1} = \beta X_t. \quad (2)$$

where all the others are fixed parameters. You have to analyze the system and:

(a) Try to detect the (possible) steady states;

(b) Try to understand under which condition(s) the steady states are stable;

(c) Show the bifurcation diagrams for different parameters, and try to tell which kind of bifurcation(s) you have encountered;

(d) Compute the main statistics of the endogenous variables (for example median, mean, standard deviation, kurtosis, ..etc...);

Please, write a report with calculations, tables, graphs and the (commented) python code(s). I do suggest you to use latex. You are expected to deliver a .pdf file of the report plus the python file(s). Please, send everything by email to Francesco Campigli and me.

Consider the following simple two-dimensional quadratic map:

$$x_{t+1} = 1 - \gamma x_t^2 + y_t,$$
$$y_{t+1} = \beta x_t,$$

where γ and β are parameters. We can write it as a difference equation $(x_{t+1}, y_{t+1}) = H_{\gamma,\beta}(x_t, y_t)$, where $H_{\gamma,\beta}$ is the 2-D map

$$H_{\gamma,\beta}(x, y) = \left(1 - \gamma x^2 + y, \beta x\right).$$

First we consider the steady states or fixed points of the map and investigate their stability. A steady state (x, y) of the map must satisfy the equations

$$1 - \gamma x^2 + y = x,$$
$$\beta x = y,$$

Substituting the second into the first equation yields

$$\gamma x^2 + (1 - \beta)x - 1 = 0$$

with solutions

$$x_{1,2} = \frac{\beta - 1 \pm \sqrt{(1-\beta)^2 + 4\gamma}}{2\gamma}$$

The steady states are

$$(x_1, y_1) = \left(\frac{\beta - 1 + \sqrt{(1-\beta)^2 + 4\gamma}}{2\gamma}, \beta x_1\right),$$
$$(x_2, y_2) = \left(\frac{\beta - 1 - \sqrt{(1-\beta)^2 + 4\gamma}}{2\gamma}, \beta x_2\right).$$

The stability of the steady states is determined by the eigenvalues of the Jacobian matrix at the steady states. Recall that a steady state is stable if all eigenvalues of the Jacobian matrix at the steady state are inside the unit circle. The Jacobian matrix $JH_{\gamma,\beta}$ of the map is given by

$$JH_{\gamma,\beta}(x, y) = \begin{pmatrix} -2\gamma x & 1 \\ \beta & 0 \end{pmatrix}.$$

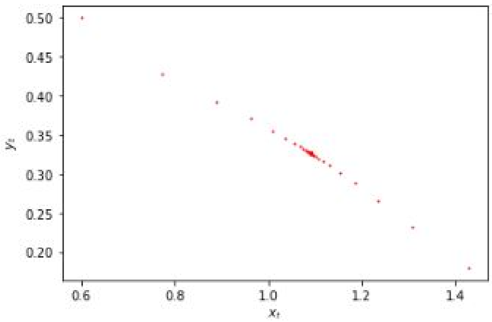
The characteristic equation determining the eigenvalues at the steady states is

$$\lambda^2 + 2\gamma x_i \lambda - \beta = 0$$

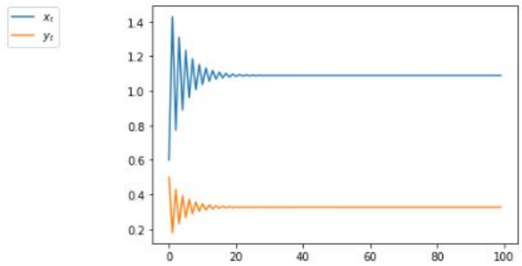
so the eigenvalues of $JH_{\gamma,\beta}(x_i, y_i)$ are

$$\lambda_i = -\gamma x_i \pm \sqrt{(\gamma x_i)^2 + \beta}.$$

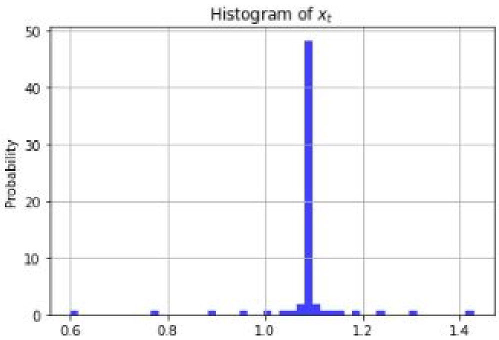
For instance, if we fix the parameters $\gamma = 1.4$ and $\beta = 0.3$. A straightforward computation shows that the eigenvalues of $JF(x_1, x_2)$ are $\lambda \simeq$ -1.92 and $\lambda \simeq 0.16$ and the eigenvalues of $JF(x_1, x_2)$ are $\lambda \simeq 3.26$ and $\lambda \simeq -0.09$. Hence, for $\gamma = 1.4$ and $\beta = 0.3$ both steady states are saddle points and therefore both are unstable.



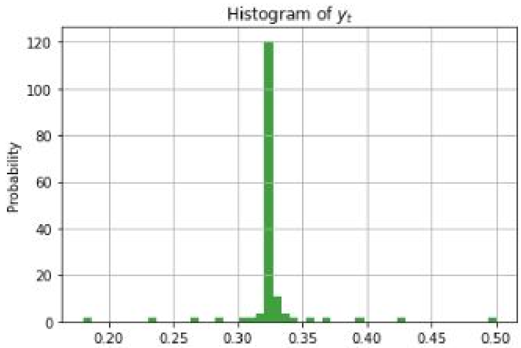
(a) $(x_t,\, y_t)$ plot



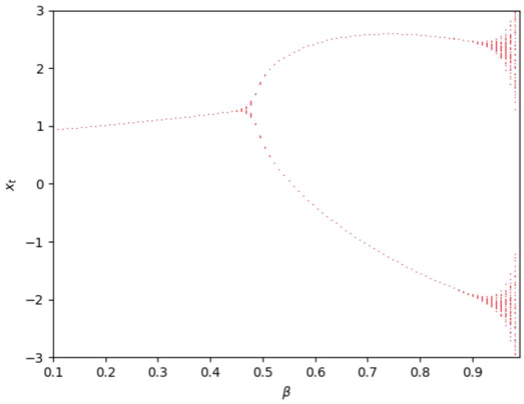
(b) Time series



(c) Histogram x_t



(d) Histogram y_t



(e) Bifourction plot