4 ISLM

4.1 ISLM: static model

The Income-Expenditure model is very simple and, among its many limitations, it considers only the goods market without incorporating the effects on aggregate demand of the interest rate (and vice versa).

We therefore present the classic IS-LM model, which combines the goods market (developed within the income-expenditure model) and the money market.

The IS (Investment-Saving) schedule

The IS is defined as the curve along which the goods market is in equilibrium. This curve is obtained starting from the Income-Expenditure model and plotting every equilibrium in the space interest rate and income. To obtain the IS, however, we must change the investment function (I), making explicit its relationship with the interest rate:

$$I = \bar{I} - br$$

where b is positive and represents the reactivity of investments to changes in the interest rate (r).
Therefore, the new expenditure line is:

$$\begin{aligned} Z &= \bar{C} + G + \bar{I} + c(T\bar{R} - T\bar{A}) - \bar{Q} + [c(1-t) - q]Y - br \\ &= \bar{Z} + \epsilon Y - br \end{aligned} \tag{10}$$

Substituting (10) into the equation and solving for Y, we get:

$$Y = z(\bar{Z} - br) \tag{11}$$

where z is the moltiplicator. Solving for r we get the IS

$$r = \frac{\bar{Z}}{b} - \frac{Y}{zb}. \tag{12}$$

Equation (12) is a line with a negative slope equal to $1/zb$, and it has intercepts on the abscissa axis equal to $\bar{Z}z$ and on the ordinate axis equal to $\bar{Z}/b$. Therefore an increase (decrease) of one of autonomous components leads to the right (left) translation of the IS. While a variation of b or of z entails a rotation of the line.

In particular, if investments are not reactive to the variations of r, that is $b = 0$, the IS is vertical while if they are infinitive reactive, at the limit $b = \infty$, the IS is flat. Furthermore, all points above (below) the IS are points where there is excess supply (demand) for goods.

The LM (liquidity-money) schedule

Let's assume the money supply is exogenous and completely in the hands of the Central Bank, which can decide how much money to put into the system. Money demand, for transactional or precautionary reasons, directly depends on income: the higher the income, the higher the value of the goods and services people is willing to buy and, therefore, the money demand.

On the other hand, agents do not hold money only as medium of exchange, but also for speculative reasons. Specifically, the wealth of agents is divided between securities and money, so securities' interest rate represents the opportunity cost of holding money. If the interest rate is close to zero, agents will be indifferent between holding money or securities. On the other hand, as the interest rate increases, the cost of holding money will increase and, therefore, there will be a reduction of money demand. For this reason, real money demand depends positively on real GDP and negatively on the interest rate:

$$L = \frac{M^d}{P} = kY - hr$$

where $h > 0$.

Equilibrium on the money market also identifies a relationship between income and the interest rate:

$$\frac{M^s}{P} = kY - hr. \tag{13}$$

the LM schedule is:

$$r = \frac{k}{h}Y - \frac{M^s}{P}\frac{1}{h}. \tag{14}$$

Equilibrium of the IS-LM static model

Given the functional shape of the two curves and the parameters, there is a unique equilibrium given by the intersection of the two curves. Mathematically it is the solution of the following system:

$$\begin{cases} r = \frac{\bar{Z}}{b} - \frac{Y}{zb} \\ \\ r = \frac{k}{h}Y - \frac{M^s}{P}\frac{1}{h}. \end{cases} \tag{15}$$

Solving by substitution, we obtain the following equilibrium values for r^E and Y^E, those for which the market for goods, that of money and that of securities are simultaneously in equilibrium:

$$\begin{cases} r^E = \frac{kz}{h+bkz}\bar{Z} - \frac{1}{h+bkz}\frac{M^s}{P} \\ \\ Y^E = \frac{hz}{h+bkz}\bar{Z} + \frac{bz}{h+bkz}\frac{M^s}{P} \end{cases} \tag{16}$$

Definying $\beta = \frac{hz}{h+bkz}$ and $\gamma = \frac{bz}{h+bkz}$:

$$\begin{cases} r^E = \frac{k}{h}\beta\bar{Z} - \frac{(1-k\gamma)}{h}\frac{M^s}{P} \\ \\ Y^E = \beta\bar{Z} + \gamma\frac{M^s}{P} \end{cases} \tag{17}$$

4.2 ISLM: discrete time dynamic model

Case a) instantaneous interest rate adjustment

Let's assume the following hypotheses:

- production slowly adjusts to excess demand:
  $\Delta Y_{t+1} = \phi(Y_t^d - Y_t)$
  where ϕ is the speed of adjustment;
- the interest rate adjusts instantly, ie the money market is always in equilibrium:
  $r_t = \frac{k}{h}Y_t - \frac{1}{h}\frac{M}{P}$

Considering these two hypotheses, the dynamics is represented by the following difference equation:

$$\begin{aligned} \Delta Y_{t+1} &= \phi(Y_t^d - Y_t) \\ Y_{t+1} &= \phi Y_t^d + (1-\phi)Y_t \\ &= \phi[\bar{Z} + \epsilon Y_t - br_t] + (1-\phi)Y_t \\ &= \phi\left[\bar{Z} + \frac{b}{h}\frac{M}{P}\right] + \left[1 - \phi\left(1 - \epsilon + \frac{kb}{h}\right)\right] Y_t \end{aligned} \tag{18}$$

The unique equilibrium is stable iff:

$$-1 < \left[1 - \phi\left(1 - \epsilon + \frac{kb}{h}\right)\right] < 1.$$

Let's study the two situation separately.

$$\begin{aligned} \left[1 - \phi\left(1 - \epsilon + \frac{kb}{h}\right)\right] &< 1 \\ -\phi\left(1 - \epsilon + \frac{kb}{h}\right) &< 0 \end{aligned} \tag{19}$$

which is always veryfied, given that ϕ and $\left(1 - \epsilon + \frac{kb}{h}\right)$ are both positive.

$$\begin{aligned} -1 &< \left[1 - \phi\left(1 - \epsilon + \frac{kb}{h}\right)\right] \\ \phi\left(1 - \epsilon + \frac{kb}{h}\right) < 2 & \\ \phi &< \frac{2}{\phi\left(1 - \epsilon + \frac{kb}{h}\right)} = \hat{\phi} \end{aligned} \tag{20}$$

If the speed of adjustments is low enough, that is less than $\bar{\phi}$, the steady state is stable.

Moreover, the convergence to equilibrium may be *monotonic* or with *oscillations*. We get the former when:

$$0 \leq \left[1 - \phi\left(1 - \epsilon + \frac{kb}{h}\right)\right] < 1.$$

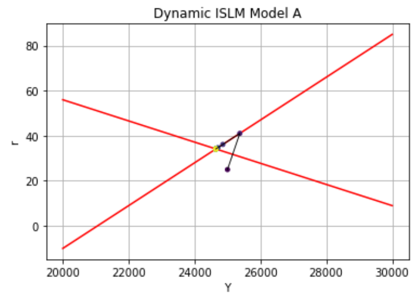

Figure 3: Monotonic Convergence: case a

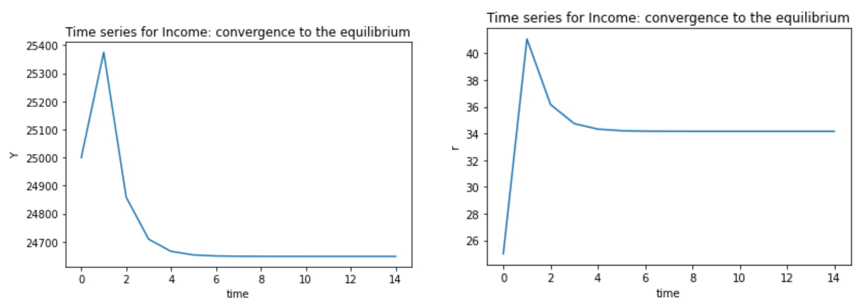

Figure 4: Convergence of Income and Interest Rate

that is for $\phi \leq \frac{1}{1-\epsilon+\frac{kb}{h}} = \frac{\hat{\phi}}{2}$.
while we get the latter when:

$$-1 < \left[1 - \phi\left(1 - \epsilon + \frac{kb}{h}\right)\right] < 0.$$

Therefore, there is an oscillatory convergence when

$$\frac{\hat{\phi}}{2} \leq \phi < \hat{\phi}.$$

It is worth noting the ϕ plays a key role only in the dynamics of this model. In figures (**??**) and (6), we report the two kind of convergence

Case b) slugghish interest rate adjustment

Let's assume the following hypotheses:

- production slowly adjusts to excess demand:

  $\Delta Y_{t+1} = \phi(Y_t^d - Y_t)$

  where ϕ is the speed of adjustment;

- the interest rate does not adjusts instantly, it depends on the excess of money demand:

  $\Delta r_{t+1} = \psi(L_t - \frac{M^s}{P}) = \psi(kY_t - hr_t - \frac{M}{P})$

Where both ϕ and ψ are the speeds of adjustment in goods and money market, respectively.

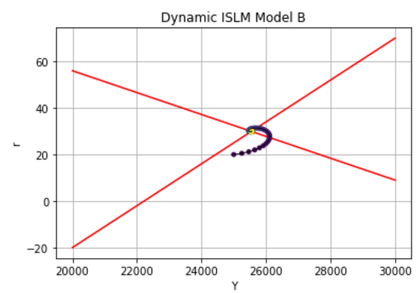

Figure 5: Oscillatory Convergence: case b

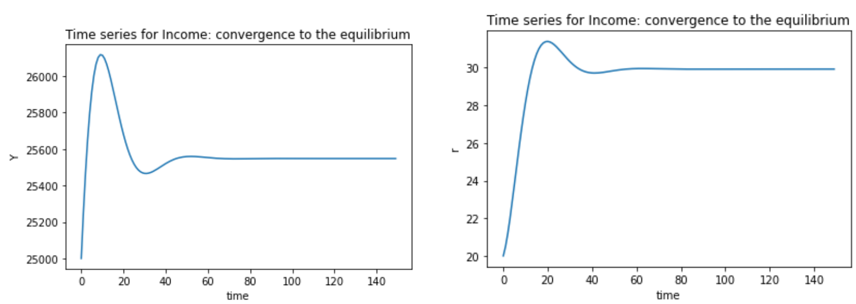

Figure 6: Convergence of Income and Interest Rate