4.3 ISLM: continuous time dynamic model without expectations

Case a: Interest rate adjusts instantaneously

Two hypotheses:

1. Production adjusts slowly: $\frac{\partial Y(t)}{\partial t} = \dot{Y} = \phi(Y^d(t) - Y(t))$
   where ϕ is the speed of adjustment;

2. the interest rate adjusts instantly, ie the money market is always in equilibrium:
   $r(t) = \frac{k}{h}Y(t) - \frac{1}{h}\frac{M}{P}$

For the sake of simplicity, let's change the notation... $Y(t) = Y$. The following system represents the two evolution of both good and money market:

$$\begin{cases} \dot{Y} = \phi[\bar{Z} + [c(1-t) - q]Y - br - Y] \\ r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{P} \end{cases} \tag{21}$$

substituting the second in the first equation, we get the following Ordinary Differential Equation (ODE):

$$\begin{aligned} \dot{Y} &= \phi[\bar{Z} + [c(1-t) - q]Y - br - Y] \\ &= \phi\left[\bar{Z} + [c(1-t) - q]Y - b\left[\frac{k}{h}Y - \frac{1}{h}\frac{M}{P}\right] - Y\right] \\ &= \phi\left[\bar{Z} + \frac{b}{h}\frac{M}{P} - \left[1 - \epsilon + \frac{bk}{h}\right]Y\right] \end{aligned} \tag{22}$$

The unique steady state of the system is exactly that of the static model. Indeed, we find it assume no changes in Y, that is $\dot{Y} = 0$: the excess of demand is zero. Therefore:

$$Y^E = \beta\bar{Z} + \gamma\frac{M}{P}$$

Its stability depends on the slope of eq. (22). Specifically, it must be negative:

$$-\phi\left[1 - \epsilon + \frac{bk}{h}\right] < 0.$$

This condition is always satisfied because both ϕ and $\left[1 - \epsilon + \frac{bk}{h}\right]$ are positive.

Case b: Interest rate adjusts slowly

Both markets adjust slowly:

$$\frac{\partial r(t)}{\partial t} = \dot{r} = \psi \left[\frac{M^d}{P} - \frac{M^s}{P} \right] \tag{23}$$

it depends on the excess of demand within the money market. Therefore, the evolution of two markets is represented by the following system:

$$\begin{cases} \dot{Y} = \phi[\bar{Z} + \epsilon Y - br - Y] \\ \dot{r} = \psi \left[kY - hr - \frac{M}{P} \right] \end{cases} \tag{24}$$

Let's rewrite it in matrix notation:

$$\begin{bmatrix} \dot{Y} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\phi(1-\epsilon) & -\phi b \\ \psi k & -\psi h \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} + \begin{bmatrix} \phi \bar{Z} \\ -\psi \frac{M}{P} \end{bmatrix}$$

The fix point may be found imposing the steady state condition, $\dot{Y} = \dot{r} = 0$. Straight forward algebra, the usual equilibrium of the static ISLM model is achieved. In order to study the stability properties of the steady state, the signs of trace and determinant must be analyzed The trace

$$-[\phi(1-\epsilon) + \psi h]$$

is negative. On the other hand, the determinant

$$\phi\psi h[1-\epsilon] + \phi b \psi k$$

is positive. This means that the steady state is always stable. In order to determine whether the steady state is a node or a spiral, we have to control for which parameters $Trace^2 \geq 4 * Determinant$. Assuming, that $\phi = \psi = 1$, if:

$$[(1-\epsilon) - h]^2 > 4bk$$

the steady state is a node, otherwise it is a spiral. In the former case there is a monotonic convergence, while in the latter it is oscillatory.