2.1 Income Expenditure Model (static)

Income Expenditure Model:

- It is the simpler macroeconomic model;
- it's static;
- prices are fixed, therefore it represents a short-run situation;
- the economic system may be closed or opened;

The starting point is the definition of GDP, as sum of expenditures:

$$Z = C + I + G + X - Q \tag{1}$$

Let's assume that the consumption function is linear. It depends on the disposable income ($Y_d = Y - \bar{TA} - tY + \bar{TR}$):

$$C = \bar{C} + cY_d = \bar{C} + c[Y - \bar{TA} - tY + \bar{TR}] \tag{2}$$

where $\bar{C}$ is the autonomous consumption, which depends on either Wealth or Price Level; c is the marginal propensity to consume, the variation of consumption for any variation of Y_d. c is the slope of the linear function and it is more than 0 but less than 1. Finally, $\bar{TA}$ represents fixed taxes, t the tax rate and $\bar{TR}$ are government transfers to households.

The import function is also assumed linear:

$$Q = \bar{Q} + q * Y$$

with $\bar{Q}$ the autonomous imports and q the marginal propensity to import. While I, G and X are assumed exogenous. Therefore, the equation (1) is:

$$Z = \bar{C} + G + \bar{I} + c(\bar{TR} - \bar{TA}) - \bar{Q} + [c(1-t) - q]Y = \bar{Z} + \epsilon Y$$

where $\bar{Z} = \bar{C} + G + \bar{I} + c(\bar{TR} - \bar{TA}) - \bar{Q}$ is the sum of autonomous components, while $\epsilon = [c(1-t) - q]$ is the slope of the expenditure function.

At the equilibrium the sum of demands is equal the sum of incomes and that of value added, that is $Z = Y$:

$$Y = \bar{Z} + \epsilon Y$$

solving for Y, we get the equilibrium:

$$Y^E = \frac{\bar{Z}}{1 - c(1-t) + q} = \bar{Z}z$$

2.2 Income Expenditure Model (dynamic)

Within the static version of the Income Expenditure model, we are interested on how equilibrium changes: the adjustments are instantanous. In order to rewrite the static version in a dynamic, we should introduce a dynamic process and therefore $time$. Moreover, it's not only relevant the existence and the uniqueness of the equilibrium but also its stability. An equilibrium is stable if, given the initial condition, it will be achieved over time. It is unstable otherwise.

Let's start, detecting the law of motion of the Income Expenditure model. Specifically, the variation of production is a function of the excess of demand:

$$\Delta Y_{t+1} = \phi(Y_t^d - Y_t) \tag{3}$$

where $\phi > 0$ and it reprensents the speed of adjustment. While Y_t^d is the sum of expenditures at time t, that is Z_t. Equation (3) is a 1D linear difference equation.

2.2.1 Steady State

The steady state of a dynamic model is a sitution in which the endougenous variable (Y) does not change over time, that is $Y_{t+1} = Y_t = Y^*$. This is true when the excess of demand is equal zero:

$$Y_t = Y_t^d = \bar{Z} + \epsilon Y_t$$

the steady state of the dynamic model is the equilibrium of the static one:

$$Y^* = Y^E = \bar{Z}z.$$

It is worth noting that the speed of adjustment does not play any role.

2.2.2 Stability

In order to analyze the stability properties, we have to rewrite equation (3) as follows:

$$\begin{aligned} Y_{t+1} &= \phi Y_t^d + (1-\phi)Y_t \\ &= \phi[\bar{Z} + \epsilon Y_t] + (1-\phi)Y_t \\ &= \phi\bar{Z} + [1-\phi(1-\epsilon)]Y_t \end{aligned} \tag{4}$$

The unique equilibrium is stable iff:

$$-1 < [1-\phi(1-\epsilon)] < 1.$$

Let's study the two situation separately.

$$\begin{aligned} [1-\phi(1-\epsilon)] &< 1 \\ -\phi(1-\epsilon) &< 0 \end{aligned} \tag{5}$$

which is always veryfied, given that ϕ and $(1-\epsilon)$ are both positive.

$$\begin{aligned} -1 &< [1-\phi(1-\epsilon)] \\ \phi(1-\epsilon) &< 2 \\ \phi &< \frac{2}{(1-\epsilon)} = \bar{\phi} \end{aligned} \tag{6}$$

If the speed of adjustments is low enough, it is less than $\bar{\phi}$, the steady state is stable.

Moreover, the convergence to equilibrium may be *monotonic* with *oscillations*. We get the former when:

$$0 \leq [1-\phi(1-\epsilon)] < 1.$$

that is for $\phi \leq \frac{1}{1-\epsilon} = \frac{\bar{\phi}}{2}$.
while we get the latter when:

$$-1 < [1-\phi(1-\epsilon)] < 0.$$

Therefore, there is an oscillatory convergence when

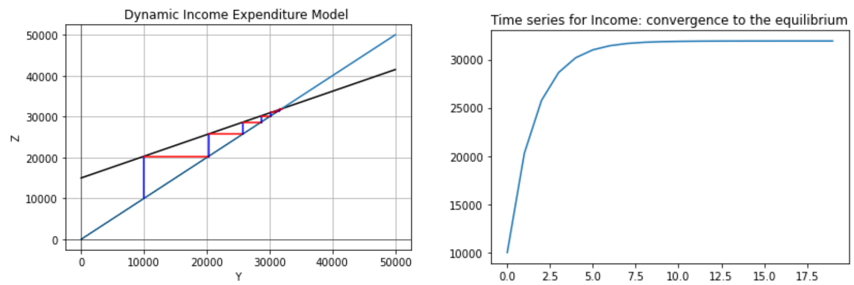

Figure 1: Monotonic Convergence

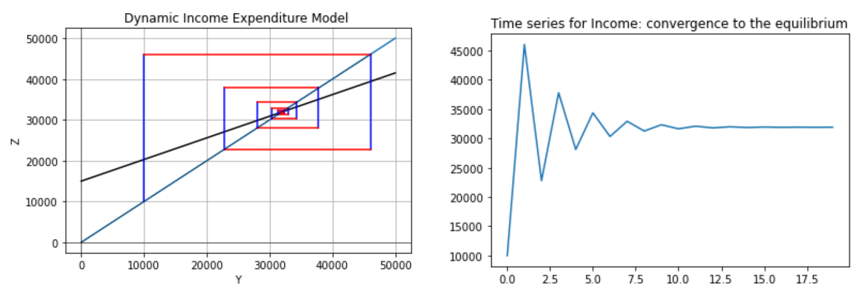

Figure 2: Oscillatory Convergence

$$\frac{\bar{\phi}}{2} \leq \phi < \bar{\phi}.$$

It is worth noting the ϕ plays a key role in the dynamics of this simple model. In figures (1) and (2), we report the two kind of convergence