A Dynamic Exchange Rate Model with Heterogeneous Agents

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- We analyze an heterogenous agent model (HAM) in which the fundamental exchange rate is endogenously determined by the real markets.

- *A more attractive alternative would be to allow for time variation in both the fundamental value of the exchange rate as well as in the beliefs about it. In the agent based literature it is sometimes assumed the fundamental value of assets evolves as a random walk[..]. In all these models, however, **the fundamental is** a time varying variable but it **still exogenously determined**. That is, **it is not connected to the real part of the economy** in any way.* (De Grauwe and Rovira, 2012 JEDC)

In finance, the **Fundamental** is the sum of the discounted future incomes generated by the asset;

However, in general, the Fundamental is a **reference value** or a **focal point** or a **belief** about the long-run value of a variable;

Therefore, it is better to talk about beliefs.

The beliefs about the future have a subjective dimension: hardly agents reach the true fundamental value and it is really unlikely that agents have the same beliefs.

The fundamental and the beliefs usually are:

- Fixed (Brock and Hommes, 1998; Naimzada and Ricchiuti,2009; De Grauwe and Rovira, 2012) vs. Time-varying (Westerhoff, 2003; Manzan and Westerhoff, 2005; De Grauwe and Grimaldi, 2006, Proaño, 2011)

In particular, among the fixed cases:

- In B-H (1998) and in DG-R (2012) there are opposite biases or pessimists/optimists;
- In N-R (2009) we assumed that the *true* fundamental value is unknown and agents have just different beliefs, that we called heterogenous fundamentals.

Recently different authors have started to endogenize the fundamental value:

- *Lengnick and Wohltmann (JEIC,2013)*: in a NK framework, the beliefs are function of the output gap;
- *Proaño* (JEBO, 2011) analyzes a two country macroeconomic model for the determination nominal exchange rate, in which the fundamental value depends on the purchasing power parity;
- *Westerhoff (DDNS, 2012)*: linked a stock market with heterogeneous speculators with a Income-Expenditure Keynesian model for a closed economy;
- *Naimzada and Pireddu (EC,2014a, C,2014b)*: in line with W(2012), they assume agents make their decisions using a linear combination between an exogenous and an endogenous value of both national income and level of stock market.

In the N-R model (2009):

$$P(t+1) = P(t) + m[n_1(F_1 - P(t)) + n_2(F_2 - P(t))]$$

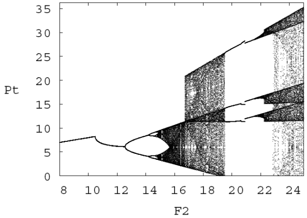

Figure : Increasing degree of heterogeneity: increasing $(F_2 - F_1)$

We combine this simple heterogenous agent model for the exchange rate with the Income-Expenditure Model for two Countries (the Absorption Model, as in Laursen and Metzler, 1950), endogeneizing F_1 and F_2.

There are two countries (Europe and US), who freely trade without restrictions and who have both a flexible exchange rate regime. As in Laursen and Metzler (1950) we assume that *'foreign exchange may be purchased and sold freely for all purposes except capital movement'*.

Foreign Exchange Market:

- as in Kirman (1998) agents act on the currency market using mechanisms developed by n **gurus**, who have developed their own mechanism to predict the future exchange rate, which is a **focal point** for the agents;
- agents join a specific guru and make decisions following her/his focal point;
- the quota of agents, who follow a specific guru, may be either fixed or variable;
- the complexity of the macro data are known for the gurus but not for the agents;

$E(t)$ is the exchange rate between the two countries at time t, expressed as *dollars per euros*.

The actual exchange rate is determined by a *market maker* who looks at the excess of demand of euros in the system.

The law of motion of the exchange rate is as follows:

$$E(t+1) = E(t) + g_3(ED(t))$$

where g_3 describes the adjustment of the market maker.

We have the following assumption for the balance of payment (BoP): *'foreign exchange may be purchased and sold freely for all purposes except capital movement'.*

The BoP depends only on the current account. The gurus estimates their belief on the exchange rate, looking at:

$$\tilde{F}(t) = \frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}.$$

that is, the value of the exchange rate for which the balance of payment is equal to zero.

Assuming there are n gurus, we suppose that, for every $j \in \{1, \dots, n\}$, guru j determines its forecasting on the exchange rate at time t, as follows:

$$\tilde{E}_j(t) = E(t) + k_j\left(\tilde{F}(t), E(t)\right) = E(t) + k_j\left(\frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}, E(t)\right)$$

where k_j is a procedure used by guru j to predict variation of the exchange rate.

The only constraint on k_j is that when guru j observes that

$$\frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)} = E(t),$$

she believes that in the future the exchange rate will not change.

Guru j is a **fundamentalist** if

$$\text{for every } (z_1, z_2) \in \mathbb{R}^2, \ \mathrm{sign}(k_j(z_1, z_2)) = \mathrm{sign}(z_1 - z_2),$$

while she is a **contrarians** if

$$\text{for every } (z_1, z_2) \in \mathbb{R}^2, \ \mathrm{sign}(k_j(z_1, z_2)) = -\mathrm{sign}(z_1 - z_2),$$

$k : \mathbb{R}^2 \to \mathbb{R}^n$ is defined, for every $(z_1, z_2) \in \mathbb{R}^2$, as $k(z_1, z_2) = (k_j(z_1, z_2))_{j=1}^n$.

The quota of agents that follow guru j at time t is:

$$\lambda_j\left(k\left(\tilde{F}(t), E(t)\right)\right),$$

where $\lambda_j : \mathbb{R}^n \to (0,1)$ is $C^1(\mathbb{R}^n)$ and, for every $x \in \mathbb{R}^n$.

We assume that, for every $x \in \mathbb{R}^n$, $\sum_{j=1}^{n} \lambda_j(x) = 1$.

The excess of demand of the agents who adhere to guru j is:

$$\lambda_j \left(k \left(\frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}, E(t) \right) \right) \gamma k_j \left(\frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}, E(t) \right)$$

where $\gamma > 0$ is a constant that transform the dollars in euros and represents the reactivity of agents.

Therefore, the dynamics of the exchange rate is:

$$E(t+1) = E(t) + g_3 \left(\sum_{j=1}^{n} \lambda_j \left(k \left(\frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}, E(t) \right) \right) \gamma k_j \left(\frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}, E(t) \right) \right),$$

where g_3 is a regular function such that $g(0) = 0$ and $g'(s) > 0$.

The demand/expenditure of European citizens in euros at time t is:

$$D_1(t) = A_1 + mpc_1 Y_1(t) + \frac{mpi_2 Y_2(t)}{E(t)} - mpi_1 Y_1(t)$$

and that of US citizens in dollars at time t is:

$$D_2(t) = A_2 + mpc_2 Y_2(t) + mpi_1 Y_1(t)E(t) - mpi_2 Y_2(t).$$

The European GDP at time $t+1$ is:

$$Y_1(t+1) = Y_1(t) + g_1(D_1(t) - Y_1(t))$$

while the US GDP at time $t+1$ is:

$$Y_2(t+1) = Y_2(t) + g_2(D_2(t) - Y_2(t))$$

where g_1 and g_2 are regular functions defined on $\mathbb{R}$ such that the first derivative is positive and $f_i(0) = 0$.

This is a short-medium-run model therefore we believe that the assumption of fixed prices is not so stringent. We will relax it, including the money market in a further work. Here, we wanted to focus on the effect of imports and exports on the exchange rate determination.

The final system is:

$$
\begin{cases}
Y_1(t+1) = Y_1(t) + g_1 \left(A_1 + mpc_1 Y_1(t) + \frac{mpi_2 Y_2(t)}{E(t)} - mpi_1 Y_1(t) - Y_1(t) \right) \\
Y_2(t+1) = Y_2(t) + g_2 \Big(A_2 + mpc_2 Y_2(t) + mpi_1 Y_1(t) E(t) - mpi_2 Y_2(t) - Y_2(t) \Big) \\
E(t+1) = E(t) + g_3 \left(\sum_{j=1}^{n} \lambda_j \left(k \left(\frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}, E(t) \right) \right) \gamma k_j \left(\frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}, E(t) \right) \right)
\end{cases} \tag{1}
$$

where the functions Y_1, Y_2 and E are assumed to be positive.

Given the mild assumptions on the functions involved in the model, it is straightforward that multiple equilibria may arise for many specifications of them. The set of steady states of the system (1) is given by

$$S = \left\{ \left(\frac{r_1(E)}{mpi_1}, \frac{r_2(E)}{mpi_2}, E \right) \in \mathbb{R}^3_{++} : E \in \Gamma \right\}$$

where

$$\Gamma = \left\{ E \in \mathbb{R}_{++} : \sum_{j=1}^{n} \lambda_j \left(k \left(\frac{r_2(E)}{r_1(E)}, E \right) \right) k_j \left(\frac{r_2(E)}{r_1(E)}, E \right) = 0 \right\}. \tag{2}$$

In particular,

$$(Y_1^*, Y_2^*, E^*) = \left(\frac{A_1}{1 - mpc_1}, \frac{A_2}{1 - mpc_2}, \frac{A_2\, mpi_2\,(1 - mpc_1)}{A_1\, mpi_1\,(1 - mpc_2)} \right) \tag{3}$$

belongs to S and is the unique steady state if, for every $(z_1, z_2) \in \mathbb{R}^2_{++}$,

$$\mathrm{sign}(k_1(z_1, z_2)) = \ldots = \mathrm{sign}(k_n(z_1, z_2)).$$

that is, it is unique if all gurus act as fundamentalists (contrarians).

Among the steady states there is always (3) which has some interesting economic properties:

1. differently from Laursen and Metzler (1950), when equilibrium is locally stable, after a small perturbation the flexible exchange rate can isolate the two economic systems driving the achievement of the new equilibrium.
2. And that an increase of income in country 1 does not lead to an increase of income in country 2, as suggested by the absorption model: the variations are absorbed by the fluctuations of the exchange rate.
3. Moreover, the exchange rate in (3) is the value that guarantees the equilibrium of the Balance of Payment (This is the only steady state having that property). Differently from Naimzada and Ricchiuti (2009) and De Grauwe and Rovira (2012), at that *fundamental* steady state if the gurus observe that the exchange rate is equal the fundamental, they believe that in the future the exchange rate will not change.
4. In a world where the determination of the exchange rate completely depends on speculative agents, it is possible that a belief around a fundamental leads to its self-fulfill.

- from a stability point of view, we find several sufficient conditions for either stability or instability;
- these conditions involve the parameters $g_1'(0)$ and $g_2'(0)$ and marginal propensities; the reactivity of agents (via γ), gurus (via k) and exchange market (via $g_3'(0)$);
- we get instability when all the gurus are contrarians;
- Similarly to Naimzada and Ricchiuti (2008, 2009), instability can also be obtained when all the gurus are fundamentalists, provided the reactivity of gurus, agents and markets are high enough.
- Finally, under the assumption that the two countries are equal and all gurus are fundamentalists, we get stability when the good market reactivity is less than twice the well-known oversimplified multiplier and all the other reactivities are low enough.

- We develop a simple heterogenous agents model in which the fundamental value of the exchange rate is endogenous, depending on the balance of payments and through it gives and receives feed-backs from the real markets.
- We overcome the exogenous determination of fundamentals, one of the most important shortcomings in the literature on heterogeneous agents models.
- We believe that further researches have to overcome some issues of our paper.
  1. even though our main aim here was to build a framework with an endogenous fundamental reducing complexity at minimum, in the foreign market we neglect the interesting case of gurus who are trend followers or noise traders.
  2. free capital transactions should be introduced to analyse the effects of the financial sector on both foreign and good markets. However, this entails a broader and more complex model with the addition of (at least) the money market (Tramontana and Ricchiuti are going in this direction).
  3. we believe that a key issue in this kind of models is the definition of time. We assumed along the paper that variations of both the exchange rate and incomes take place at the same moment. To overcome this issue, we believe that it should be studied a hybrid model in which the variations happen in different time scale.