Heterogeneous Fundamentalists and Market Maker Inventories

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Heterogeneous Fundamentalists and Market Maker Inventories:

- Recently, Naimzada and Ricchiuti (2008, 2009, 2012) developed a framework in which the source of instability resided in the interaction of **two different groups of agents with the same trading strategy** but **different beliefs** about the fundamental asset values;
- In this paper we add to this structure the Marker Maker Inventory;

A **market maker** or liquidity provider is a company, or an individual, that quotes both a buy and a sell price in a financial instrument or commodity held in inventory, hoping to make a profit on the bid-offer spread, or turn. (Radcliffe, 1997)

The double role of the Market Maker

1 - Dealer or liquidity provider:

- reacts to excess demand (supply) setting the price and accumulating/decumulating stocks in presence of excess of supply (demand);

2 - Active Investor:

- maximizes his profits by actively managing his inventory.

Similar models with two heterogeneous agents and a market maker acting both as a dealer and an active investor but with a market maker inventory are reported by Westerhoff (2003) and Zhu *et al.* (2009)

Research questions

Hence, in a framework consisting of two fundamentalists plus a market maker which actively manages his inventory:

- Does the market maker **stabilize or destabilize** the market?
- Which is the relation between the **fraction of inventory** the market maker holds from the previous period and the market stability?
- Do the different **beliefs about the fundamentals** influence the market stability in this framework?

Stylized facts

We are also trying to replicate the most important stylized facts of the asset-market:

- excess volatility
- fat tails;
- bubbles and crashes;
- volatility clustering;
- long range dependence;
- leverage effect.

Characteristics

Characteristics of the model:

- We consider a market populated by **two groups of fundamentalists** and a **market maker**;
- Agents can invest in two different assets: risky and risk free asset;
- All the investors choose their own portfolio in a way such they maximize their expected utility;
- z_s is the total fixed risky asset supply;
- $E_{i,t}(X_{t+1})$ and $V_{i,t}(X_{t+1})$ are the beliefs or forecasts about the future dividends and the conditional variance of the quantity X_{t+1} respectively.

Let's assume for agents of group i a CARA utility function:
We assume that agents have common expectations on **dividends**...

$$E_{i,t}(y_{t+1}) = E_t(y_{t+1}) = \bar{y} \tag{1}$$

...but different beliefs on **future prices**

$$E_{i,t}(X_{t+1}) = E_i(X^*_{t+1}) = F^*_i \tag{2}$$

The demand of the two groups of agents is:

$$q_{i,t} = \delta(F^*_i - P_t) \tag{3}$$

Inventory of a market maker acting as a **dealer**

1. The traders aggregate demand z_t^* at time t is equal to

$$z_t^* = n_{1,t+1} q_1 + (1 - n_{1,t+1}) q_2 + \epsilon_t, \quad with \quad n_1 + n_2 = 1, \tag{4}$$

   n_i are the agents' fractions, which can be fixed or variable; ϵ_t is the demand's error term;

2. The market maker seeks to maintain a long term desired target inventory position I_{t+1}^d

$$I_{t+1}^d = \kappa I_t + (1 - \kappa) I^d, \quad with \quad \kappa \in [0, 0.5) \tag{5}$$

   where κ is the share of inventory the market maker holds from the previous period;

3. The market maker inventory is:

$$I_{t+1} = I_{t+1}^d + (z_s - z_t^*). \tag{6}$$

4. Substituting (5) in (6):

$$I_{t+1} = \kappa I_t + (1 - \kappa) I^d + (z_s - z_t^*) \tag{7}$$

The Market Maker

- Given our assumptions we model the market excess demand ED_t for the risky asset in trading period $t+1$ as
$$ED_t = z_t^* + I_{t+1}^d - z_s \tag{8}$$
- On the other side the market maker adjusts the price so that the return is an increasing function of the market excess demand:
$$P_{t+1} = P_t + P_t\gamma[ED_t] = P_t + P_t\gamma[z_t^* + I_{t+1}^d - z_s]. \tag{9}$$
where $\gamma > 0$ is the sensitivity of the market maker to the excess demand.

The dynamic model

The asset price and the inventory dynamics are thus determined by the following random discrete non-linear dynamical system of equations:

$$\begin{cases} P_{t+1} = P_t[1 + \gamma[n_1(F_1 - P_t) + n_2(F_2 - P_t) + \kappa I_t + (1-\kappa)I^d - z_s]] + \epsilon_t \\ I_{t+1} = \kappa I_t + (1-\kappa)I^d + [z_s - [n_1(F_1 - P_t) + n_2(F_2 - P_t)]] + \epsilon_t \end{cases} \tag{10}$$

Dynamic Analysis of the deterministic model: fixed points

Solving the bi-dimensional map (10) for its roots, we obtain two steady state points: (11) and (12).

$$(P_1^*, I_1^*) = \left(0, \frac{z_s - G}{1-\kappa}\right) \tag{11}$$

and

$$(P_2^*, I_2^*) = \left(G - z_s + \frac{(1-\kappa)I^d}{1-2\kappa}, \frac{2(1-\kappa)I^d}{1-2\kappa}\right) \tag{12}$$

with $G = n_1 F_1 + n_2 F_2$ representing the weighted average of the two fundamentals

Dynamic Analysis: Stability

- Jacobian Matrix

$$J = \begin{bmatrix} 1 + \gamma(G - 2P_t + \kappa I_t + (1-k)I^d - z_s) & \gamma\kappa P_t \\ 1 & \kappa \end{bmatrix} \quad (13)$$

- Jury's conditions for stability (Jury, 1974), we have the following necessary and sufficient condition for local stability of the second fixed point:

$$\begin{cases} \gamma((\kappa-1)I^d + (2\kappa-1)G + (1-2\kappa)z_s) < 0 \\ (2\kappa-1)\left[2-(4\kappa-2)\kappa + G\gamma(4\kappa^2-1) + (1+2\kappa)\gamma\left((\kappa-1)I^d + (1-2\kappa)z_s\right)\right] < 0 \\ \frac{\kappa[2\kappa-1+2\gamma((1-2\kappa)G+(1-\kappa)I^d+(2\kappa-1)z_s)]}{2\kappa-1} < 1 \end{cases} \quad (14)$$

Numerical Analysis: Calibration with fixed **fractions**

In order to study the local stability properties of the equilibrium points conveniently, we calibrate the model according to the characteristics highlighted in our framework and replicate the parameters reported by Zhu et al. (2009).

γ	z_s	κ	n_1	n_2	F_1	F_2	I^d
0.179	1	0.15	0.5	0.5	2	3	10

Table : Parameter settings for the simulations

- We analyze firstly the model with fixed fractions and then that with a switching mechanisms *a la* Brock and Hommes.
- Let's see..

Conclusions

- **The higher the degree of heterogeneity between fundamentalists, the more responsive is the market stability to lower levels of reactivity**.
- The **market maker** strongly **destabilizes the market** as in Zhu et al. (2009): when agents opinions consistently diverge, increasingly lower shares of inventory quotas held by the market maker strongly affect the market stability.
- Buffeted with dynamic noise, this model also replicates some important stylized facts of financial markets: in particular volatility clustering, price increments and excess volatility. We do not find fat tails, the two beliefs keep the range between the two prices narrow.

Further improvements

- Further improvements to our behavioural financial model may consist in analyzing a case when more actors with a long memory are introduced in the model.
- Adding a chartist;
- Control for market distortions and price volatility: introducing **price limiters** as in Westerhoff (2003).
- Incorporate different market maker structures as in Anufriev and Panchenko (2009) and analysing how time series behaviour varies.