# Heterogeneous beliefs and routes to chaos in a simple asset pricing model

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- Zeeman 1974;
- Goldman (1980);
- Frankle and Froot (1986, 1990);
- Day and Huang (1990);
- Chiarella (1992);
- Brock and Hommes (1997, 1998);
- Farmer (2002), Joshi (2002);
- and so on...

Main Elements:

- Group of Agents with different strategies (Fundamentalists, Chartists, Noise traders, Bias traders, Naive)
- Fixed quotas vs. Evolutionary switching mechanisms
- Dynamical systems
- Walrasian Auctioner vs. Market Maker

BH1998, 'Heterogeneous beliefs and routes to chaos in a simple asset pricing model', JEDC Vol.22, Issue 8-9

- It is a financial market application of BH1997
- There is bounded rationality
- Agents select their strategy rule based upon recent relative performance
- It is a stylized model, to some extent analytically tractable
- Really good: it is formulated in terms of deviation from a RE benchmark

- Agents can invest in risk free or in a risky asset
- The risk free is perfectly elastically supplied and pays an interest $r$. While the risky asset pays and uncertain dividend;
- Agents are myopic mean-variance maximizers so the demand per trader of type $h$ for the risky asset is:

$$z_{ht} = \frac{E_{ht}[p_{t+1} + y_{t+1} - (1+r)p_t]}{a\sigma^2}$$

  where $E_{ht}$ are belief of trader type $h$, and $y_{t+1}$ is the dividend, and $a$ is the risk aversion parameter.
- $z^s$ is the constant supply
- $n_{ht}$ is the quota of group $h$ at time $t$.

- therefore, the equilibrium between demand and supply is

$$\sum_{h=1}^{H} n_{ht} \frac{E_{ht}[p_{t+1} + y_{t+1} - (1+r)p_t]}{a\sigma^2} = z^s$$

- BH98 assumes that $z^s = 0$, therefore the Walrasian market clearing price satisfies:

$$(1+r)p_t = \sum_{h=1}^{H} n_{ht} E_{ht}[p_{t+1} + y_{t+1}]$$

- in an homogeneous world:

$$p_t^* = \sum_{k=1}^{\infty} \frac{E_t[y_{t+k}]}{(1+r)^k}$$

assuming that the dividend is equal for each period

$$p^* = \sum_{k=1}^{\infty} \frac{\bar{y}}{(1+r)^k} = \frac{\bar{y}}{r}$$

Heterogenous Beliefs:

- in a heterogeneous world traders can deviate from the fundamental price:

$$x_t = p_t - p_t^*$$

- and beliefs are:

$$E_{ht}[p_{t+1}] = E_t[p_{t+1}^*] + f_h(x_{t-1}, ....x_{t-L}) = E_t[p_{t+1}^*] + f_{ht}$$

  where $f_h$ represents a strategy rule.

- therefore

$$(1 + r)x_t = \sum_{h=1}^{H} n_{ht} f_{ht}$$

- Evolutionary selection strategies follow a discrete choice approach

$$n_{ht} = \frac{exp(\beta U_{h,t-1})}{\sum^{H_{h=1}} U_{h,t-1}}$$

- in BH the evolutionary fitness is given by the realized excess returns (profits)

$$U_{ht} = (p_t + y_t - Rp_{t-1}) \frac{E_{ht}[p_t + y_t - Rp_{t-1}]}{a\sigma^2} + \omega U_{h,t-1}$$

  where $\omega$ is a memory parameter. Let's assume that $\omega = 0$.

- therefore:

$$U_{ht} = (x_t - Rx_{t-1}) \frac{f_{h,t-1} - Rx_{t-1}}{a\sigma^2}$$

Let's assume that:

$$f_{ht} = g_h x_{t-1} + b_n$$

where $g_h$ is a trend parameter and $b_n$ a bias parameter. Therefore

1. fundamentalist: $g_h = b_n = 0$
2. trend-followers-chartists: $b_n = 0$
3. biased belief: $g_h = 0$