Asset Price Dynamics with Heterogeneous Beliefs and Local Network Interactions

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Why Networks in Asset Pricing?

In this paper Panchenko, Gerasymchuk & Pavlov (2013, *JEDC*) investigate the effects of network topologies on asset price dynamics. They introduce network communications into the BH (1998) model with heterogeneous beliefs. The performance information is available to agents only **locally** – through their own experience and the experience of directly connected neighbours.

Empirical motivation:

- Fund managers rely on peer conversations (Shiller & Pound, 1989; Hong et al., 2005)
- Retail investors follow neighbours and church friends (Hong et al., 2004)
- Board connections through shared education networks affect portfolio returns (Cohen et al., 2008)

Four Network Topologies

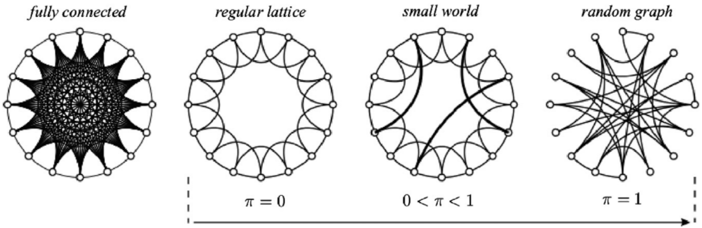

Fig. 1. Network topologies (adapted from Watts and Strogatz, 1998). π indicates a link rewiring probability.

K = degree, N = total nodes, π = rewiring probability. Fully connected: all nodes linked. Regular lattice ($\pi = 0$): fixed K neighbours, high clustering, long paths. Small world ($0 < \pi < 1$): high C, short L. Random graph ($\pi = 1$): low C, short L.

Information Transmission in Networks

A key concept running through the entire paper:

Information latency

How long it takes for news about an alternative strategy's performance to reach an agent through the network. The slower the transmission, the longer agents are "stuck" with their current strategy – unable to switch even if it is performing poorly.

Information transmission speed depends on three network characteristics:

- **Average degree** k: more neighbours $\Rightarrow$ more chances to observe an alternative strategy
- **Characteristic path length** L: shorter paths $\Rightarrow$ information travels faster across the network
- **Clustering coefficient** C: if your neighbours all know each other, they likely share the same strategy $\Rightarrow$ your neighbourhood is **less informative** even if large

The main result in one sentence

Higher latency $\Rightarrow$ chartists dominate longer $\Rightarrow$ markets destabilise at **lower** β $\Rightarrow$ larger price swings.

Regular Lattice ($\pi = 0$)

Each node is connected to exactly K nearest neighbours on a ring. In this paper $K = 4$ (baseline).

- **High clustering** C: your neighbours are also connected to each other
- **Long characteristic path length** L: to reach a distant node you must traverse many steps
- Information travels **slowly** – it must diffuse hop by hop around the ring
- Represents a world of **tight local communities** with no shortcuts

In the model

Agents only learn from their 4 immediate neighbours. If all neighbours share the same strategy, the agent is stuck and **cannot switch**.

Small World (Watts–Strogatz, $0 < \pi < 1$)

Start from the regular lattice. With probability π, rewire each link to a randomly chosen node (no self-loops, no double links).

- Even a **small** π dramatically reduces L while keeping C high
- **High clustering** (local communities survive) + **short paths** (a few long-range shortcuts suffice)
- This is the "six degrees of separation" phenomenon
- Realistic model for **social and financial networks**: friendship, corporate boards, fund managers

In the model

$\pi = 0.01$ (with $N = 1000$). A handful of rewired links creates global shortcuts, accelerating information transmission relative to the lattice.

Random Graph ($\pi = 1$)

All links are rewired at random: the result is **similar** to an Erdős–Rényi graph, but not identical.

- **Low clustering** C: neighbours are unlikely to know each other
- **Short path length** L: information reaches any node quickly
- Degree distribution is concentrated around K (unlike ER, where it is Poisson with true degree variability)
- In WS with $\pi = 1$ each node keeps exactly K links; in ER the degree varies across nodes

In the model

Unclustered neighbourhoods make the system analytically tractable: a low-dimensional approximation for the fraction of chartists can be derived. The authors note this network is "similar to" ER – not identical.

Watts–Strogatz: Interpolating Between Regimes

By varying $\pi \in [0, 1]$ and K, the model continuously interpolates between three regimes:

- $\pi = 0$: **regular lattice** – high C, high L
- $\pi \approx 0.01$–0.1: **small world** – high C, low L ← sweet spot
- $\pi = 1$: **random graph** – low C, low L

As π rises, L collapses quickly (a few long-range shortcuts suffice), while C falls much more slowly (local triangles survive). This is why the small-world regime is wide.

Increasing K (average degree) has a similar effect: more connections ⇒ faster information diffusion ⇒ dynamics closer to the fully connected benchmark.

The Brock–Hommes (1998) Model

BH1998, *Heterogeneous beliefs and routes to chaos in a simple asset pricing model*, JEDC Vol.22

- Agents can invest in a risk-free asset (gross return $R = 1 + r$) or a risky asset (stochastic dividend y_t)
- Agents are myopic mean-variance maximizers; demand of type h:

$$z_t^h = \frac{E_{t-1}^h[p_{t+1} + y_{t+1} - Rp_t]}{a\sigma^2}$$

- Zero outside supply $\Rightarrow$ Walrasian clearing:

$$Rp_t = \sum_{h=1}^{H} n_{t-1}^h \, E_{t-1}^h[p_{t+1} + y_{t+1}]$$

Fundamental Price and Heterogeneous Beliefs

- Fundamental price: $p^* = \bar{y}/r$
- Deviation from fundamental: $x_t = p_t - p^*$
- Belief rule:
$$E^h_{t-1}[p_{t+1} + y_{t+1}] = b^h + p^* + \bar{y} + g^h x_{t-1}$$
where b^h is a bias and g^h an extrapolation parameter
- **Fundamentalists**: $b^h = 0$, $g^h = 0$
- **Chartists**: $b^h = 0$, $g^h > 0$

Evolutionary Fitness and Strategy Switching

- Evolutionary fitness (realized profit net of costs):

$$U_t^h = (p_t + y_t - Rp_{t-1})\, z_{t-1}^h - c^h$$

- In BH, every agent observes U_t^h for **all** types; fractions follow a logit:

$$n_t^h = \frac{\exp(\beta\, U_t^h)}{\sum_\ell \exp(\beta\, U_t^\ell)}$$

- β = intensity of choice: how strongly agents react to past performance differences
- Price dynamics (two-type, $b^h = 0$): $x_t = \frac{g}{R}\, n_{t-1}\, x_{t-1}$

From BH to Local Interactions

$$E^h_{t-1}[p_{t+1} + y_{t+1}] = b^h + p^* + \bar{y} + g^h(p_{t-1} - p^*)$$

b^h is a constant bias and g^h an extrapolation parameter; $b^h = 0$ throughout. Fundamentalists: $g^h = 0$. Chartists: $g^h > 0$.

The key modification:

- Agents are located on network nodes and observe U^h_t only for **directly connected** neighbours
- They cannot observe the performance of types adopted by agents two or more links away
- If surrounded only by same-type neighbours $\Rightarrow$ agent **cannot switch** (no alternative observed)
- If at least one neighbour differs $\Rightarrow$ agent compares and switches via logit Δ_t

Lock-in and Corner Steady States

- Under local interactions, fractions no longer follow the global logit of BH
- Two **corner steady states** emerge:
  - $E_0 = (0, 0)$: all agents fundamentalists
  - $E_1 = (0, 1)$: all agents chartists
- If one type takes over completely, no agent can observe the alternative $\Rightarrow$ **lock-in**
- To prevent permanent lock-in in simulations: two "die-hard" agents (one per type) placed on opposite sides of the network

Temporal Flow

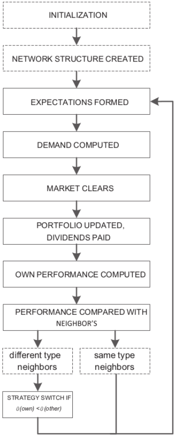

Fig. 3. Temporal flow.

Proposition 1: Steady States and Stability (two-type, random graph)

Three possible regimes depending on the extrapolation parameter g and intensity of choice β:

- **Weak trend** ($g < R$): only fundamental steady states exist. For low β, the interior state $E_\diamond$ is stable – price converges to p^*. As β rises, a **transcritical bifurcation** occurs: $E_\diamond$ collides with the corner state E_1 (all chartists) and ceases to exist
- **Intermediate trend** ($R \leq g < 2R$): for $\beta < \beta^*$, $E_\diamond$ is stable. At β^*, a **pitchfork bifurcation** – $E_\diamond$ loses stability and two non-fundamental steady states $E_\pm$ emerge (price persistently above or below p^*). At β^{**}, a **Neimark–Sacker bifurcation** – $E_\pm$ lose stability
- **Strong trend** ($g \geq 2R$): the fundamental steady state is always unstable; non-fundamental states are stable only for low β

Key difference from BH

$\beta^*_{\mathrm{RG}} < \beta^*_{\mathrm{BH}}$ and $\beta^{**}_{\mathrm{RG}} < \beta^{**}_{\mathrm{BH}}$: instability sets in **earlier** under local interactions. Both thresholds increase with k and converge to BH as $k \to \infty$.

Low-Dimensional Approximation: Random Graph

Why the random graph? Because its unclustered neighbourhoods make the system **analytically tractable**.

Since neighbours of a node are unlikely to be connected to each other, by a symmetry argument we can drop individual agent indices and track only the **aggregate fraction of chartists** n_t:

$$n_t = n_{t-1}^{k+1} + \left[1 - n_{t-1}^{k+1} - (1 - n_{t-1})^{k+1}\right] \Delta_t$$

- First term: chartists who are **surrounded only by chartists** – they cannot switch
- Second term: agents who **can** switch – they do so with probability Δ_t
- As $k \to \infty$: $n_t \to \Delta_t \Rightarrow$ recovers the original BH logit

This single equation replaces the full agent-level simulation for the random graph – it is the key analytical tool behind Proposition 1.

Steady States: Map $f(n)$

At a fundamental steady state ($x = 0$), the fraction of chartists n must satisfy $n = f(n)$, where:

$$f(n) = n^{k+1} + \left(1 - n^{k+1} - (1-n)^{k+1}\right)\Delta$$

and $\Delta = 1/(1 + \exp(\beta c))$ is the BH steady-state fraction of chartists.

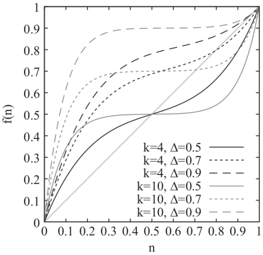

Fig. 4. Dependence of map $f(n) = n^{k+1} + (1-n^{k+1}-(1-n)^{k+1})\Delta$, on k and Δ.

Fixed points are where the map crosses the diagonal. Three always exist: two corner states $E_0 = 0$ and $E_1 = 1$ (lock-in) and one interior state $E_\diamond$ – but $E_\diamond \geq \Delta$: **more chartists persist** than in BH at the same β. As k increases the map flattens toward the diagonal and $E_\diamond \to \Delta$.

Bifurcation Values as a Function of k

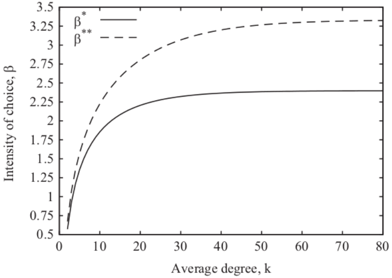

Fig. 5. Bifurcation values of β for given k, $g = 1.2$, $R = 1.1$, $c = 1$.

Both β^* and β^{**} increase with k and converge to the original BH values. For $k = 4$: $\beta^* = 1.07$, $\beta^{**} = 1.35$ – well below BH values of 2.40 and 3.33.

The Neimark–Sacker Bifurcation

The Neimark–Sacker (NS) bifurcation is the **discrete-time analogue** of the Hopf bifurcation in continuous-time systems.

- Occurs when a pair of complex conjugate eigenvalues of the Jacobian **crosses the unit circle**
- A stable fixed point loses stability and gives birth to an **invariant closed curve** in the state space
- On this curve the dynamics can be **quasi-periodic** (the orbit densely fills the curve) or **periodic** (the orbit closes after finitely many steps), depending on parameters
- For slightly larger β, the largest Lyapunov exponent becomes positive $\Rightarrow$ **weakly chaotic** dynamics

Economic interpretation

After the NS bifurcation, prices no longer converge to any fixed level – they oscillate in a complicated, irregular pattern. Chartists periodically dominate, creating bubbles; then fundamentalists take over and prices crash back. The cycle repeats but **never exactly**.

Bifurcation Diagrams: All Four Networks

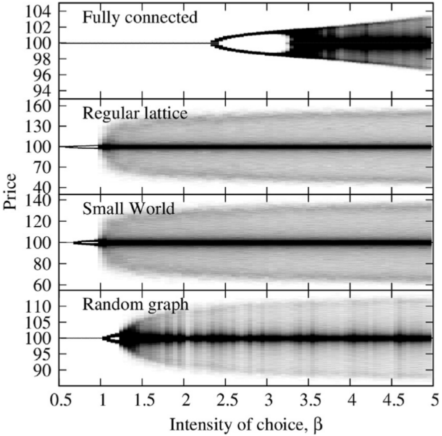

Fig. 6. Bifurcation diagrams for various network topologies.

Bifurcation Diagrams: Fixed L and Fixed C

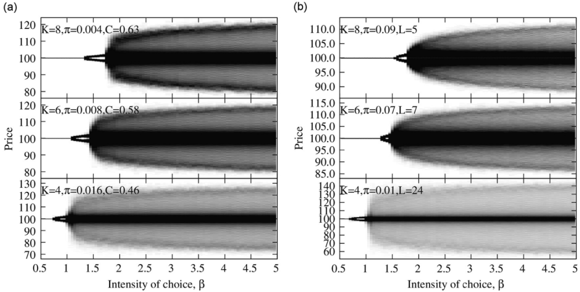

Fig. 7. Bifurcation diagrams for varying K, π, L and C. (a) Fixed $L=17$. (b) Fixed $C=0.48$.

A natural question: is there a **single network statistic** that fully predicts bifurcation values and price amplitudes? The authors fix in turn:

- L = characteristic path length: average shortest distance between any two nodes
- C = clustering coefficient: probability that two neighbours of a node are also connected to each other

Bifurcation diagrams keep changing even when k, L or C is held fixed individually $\Rightarrow$ **no single structural measure** fully captures information latency. A "clustering-adjusted degree" is left for future work.

Informational Efficiency

Informational efficiency can be measured by comparing price volatility with fundamental dividend volatility (Shiller, 1981). Under the EMH, price variance should equal fundamental variance.

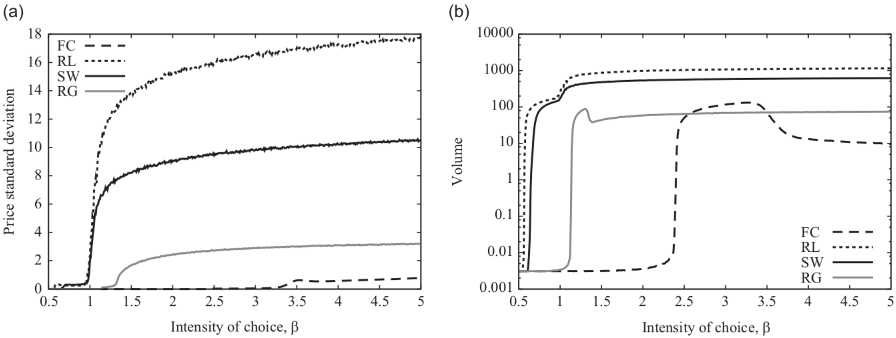

Fig. 10. Measures of market information inefficiency. (a) Standard deviation of price. (b) Average traded volume.

Return Distribution: Skewness and Kurtosis

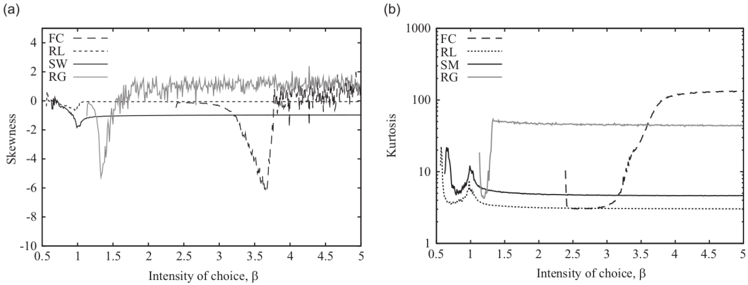

Fig. 11. (a) Skewness and (b) Kurtosis of returns.

Skewness is close to zero for all networks; SW returns are slightly negatively skewed. Kurtosis under SW ≈ 8, closest to empirical fat tails observed in financial returns.

Autocorrelations and Volume–Volatility

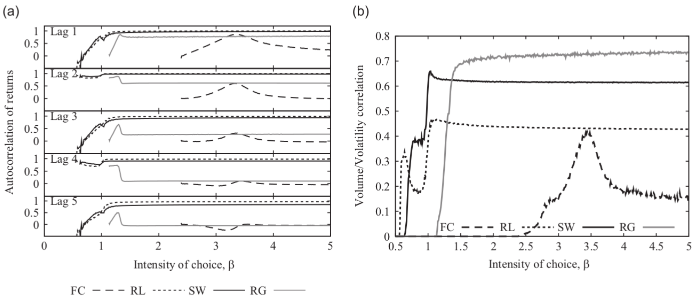

Fig. 12. Properties of returns (a) Autocorrelations of returns and (b) Volume/volatility correlations.

RL and SW produce high return autocorrelations at all lags. FC and RG converge to zero after lag 2. Autocorrelations of *squared* returns persist for RL and SW ⇒ volatility clustering consistent with GARCH-type stylized facts.

Summary Table

Table 1
Characteristics depending on the network in increasing order left to right. The characteristics depending on values of β are compared at fixed β: $\beta = 4$ for the two-type model and $\beta = 85$ for the four-type model. *Note*: $A \approx B$ indicates that there is no clear ranking between A and B, *neg* stands for negative values.

Characteristic	Two-type model	Four-type model
Latency in information transmission	FC RG SW RL	FC RG SW RL
$1/\beta$ of the primary bifurcation	FC RG SW RL	FC RG SW $\approx$ RL
Length of instability interval	FC RG SW RL	FC RG SW $\approx$ RL
Amplitude of price fluctuation	FC RG SW RL	RL SW FC RG
Std. deviation of price	FC RG SW RL	RL SW FC RG
Average trading volume	FC RG SW RL	RL SW FC RG
Skewness of returns	SW RL FC RG	RL SW FC RG
Kurtosis of returns	RL SW RG FC	SW RL FC RG
Autocorrelations of returns	FC RG SW RL	inconclusive
Volume/volatility correlations	FC RL SW RG	FC (*neg*) SM RL RG
Autocorrelations of squared returns	FC RG SW RL	FC RG RL SW

Beyond These Four: Other Network Topologies

The four topologies studied in this paper are a useful starting point, but real networks are richer:

- **Scale-free networks** (Barabási–Albert, 1999): links form via **preferential attachment** – new nodes connect to already well-connected nodes. Degree distribution follows a **power law**: a few hubs, many peripheral nodes. Common in the web, citation networks, interbank markets
- **Core-periphery networks**: a dense, highly connected core and a sparse periphery. Typical structure of **financial systems** (Fricke & Lux, 2015)
- **Multiplex networks**: agents interact on **multiple layers** simultaneously (e.g. trading network + information network + ownership network)
- **Weighted networks**: links carry intensity (trading volume, frequency of communication) not just presence/absence
- **Endogenous networks**: topology **co-evolves** with agent behaviour – who you talk to depends on how well your strategy is performing

Each topology implies different information transmission speeds, different lock-in risks, and potentially different bifurcation structures.

Conclusions

- The BH (1998) model is expanded by introducing **local information exchange** via communication networks
- A low-dimensional system is derived for the two-type model with a random graph; stability results are proved analytically
- Other topologies are studied by simulation: stability regions depend on network parameters
- In the two-type model, higher latency (RL $>$ SW $>$ RG $>$ FC) $\Rightarrow$ earlier instability, larger price swings, more volatility clustering
- No single network statistic (k, L, or C) alone characterises information transmission
- In the four-type model, latency produces **qualitatively different** dynamics – not just a quantitative shift