Beyond BH98: Profits, Paradoxes, and Learning in Asset Markets

Giorgio Ricchiuti

UNIVERSITÀ DEGLI STUDI FIRENZE

UNIVERSITÀ CATTOLICA del Sacro Cuore
CLE
Complexity Lab in Economics

Motivation

- Fundamentalists are a standard type in models with Heterogeneous Interacting Agents (HIAs)
- Fundamentalists act according to information (or beliefs) about the value of a variable
- Their expectation is
  - consistent if everyone is fundamentalist
  - not necessarily consistent when HIAs are assumed

Questions:

1. Does it matter if expectations are non-consistent with 'reality'?
2. If no, does it make sense for agents to pay to be fundamentalists?

Overview

- Study the baseline version of Brock and Hommes (1998)
- Asset pricing model with two types:
  - fundamentalists (costly strategy)
  - chartists (or trend followers)
- Focus on profits (change in wealth) of strategies
- Counterintuitive insights or 'paradoxes':
  1. Fundamentalists gain less on average
  2. Fundamentalists' profits are higher when price is far from the fundamental value
  3. Decreasing marginal profits with decreasing costs

Connection to what we already know

- Recall the BH98 framework: agents switch between strategies based on **past realized profits**
- The intensity of choice β governs how fast this switching occurs
- For high enough β and g, the model generates non-trivial dynamics: cycles, chaos

- Today we ask a different question: not *what dynamics emerge*, but *who actually earns what* along those dynamics

- The answers turn out to be **counterintuitive** — and economically important

Asset pricing model with two types

- Agents are mean variance wealth maximisers
- Two types of strategies: fundamentalist and trend following
- Two types of assets:
  - Risk free, perfectly elastically supplied, paying interest R
  - Risky pays stochastic (IID) dividend y_t and is sold at price p_t
- Assuming zero supply of outside shares, define as the *fundamental* value of the asset:

$$p^* = y/(R-1) \tag{1}$$

Price deviations and expected profits

- Let $x_t = p_t - p^*$ denote the deviation from the fundamental value
- Agents who pay C have access to the fundamental value and expect $x_t = 0$
- Trend followers expect that x_t follows a trend g compared to its value in the previous period
- Expected profits of fundamentalist and trend following strategies are

$$\pi_{f,t} = \frac{1}{a\sigma^2} R x_{t-1}(R x_{t-1} - x_t) - C, \tag{2}$$

where a is assumed risk aversion and σ^2 is the excess return variance, and

$$\pi_{c,t} = \frac{1}{a\sigma^2}(x_t - R x_{t-1})(g x_{t-2} - R x_{t-1}) \tag{3}$$

Reading the profit equations

- Both profit equations share the same structure:

$$\pi_{h,t} \propto \underbrace{(x_t - Rx_{t-1})}_{\text{realized price change}} \times \underbrace{(f_{h,t-1} - Rx_{t-1})}_{\text{forecast error relative to no-trade}}$$

- A strategy is profitable when its **forecast is on the right side** of the realized price change

- Fundamentalists always forecast $x_t = 0$: they profit when the price *returns* toward the fundamental
- Chartists forecast gx_{t-2}: they profit when the price *continues* in the direction of the trend

- The two strategies are essentially **betting against each other** on the direction of the next price move

Dynamics

Assuming a logit framework with scale parameter $1/\beta$, the fractions of fundamentalists and trend followers in each period are given by

$$n_{f,t} = \frac{e^{\beta \pi_{f,t}}}{e^{\beta \pi_{f,t}} + e^{\beta \pi_{c,t}}} \tag{4}$$

and

$$n_{c,t} = \frac{e^{\beta \pi_{c,t}}}{e^{\beta \pi_{f,t}} + e^{\beta \pi_{c,t}}} \tag{5}$$

with

$$m_t = n_{f,t} - n_{c,t} = \tanh\left[\frac{\beta}{2}\left(\frac{g x_{t-2}(R x_{t-1} - x_t)}{a\sigma^2} - C\right)\right], \tag{6}$$

and

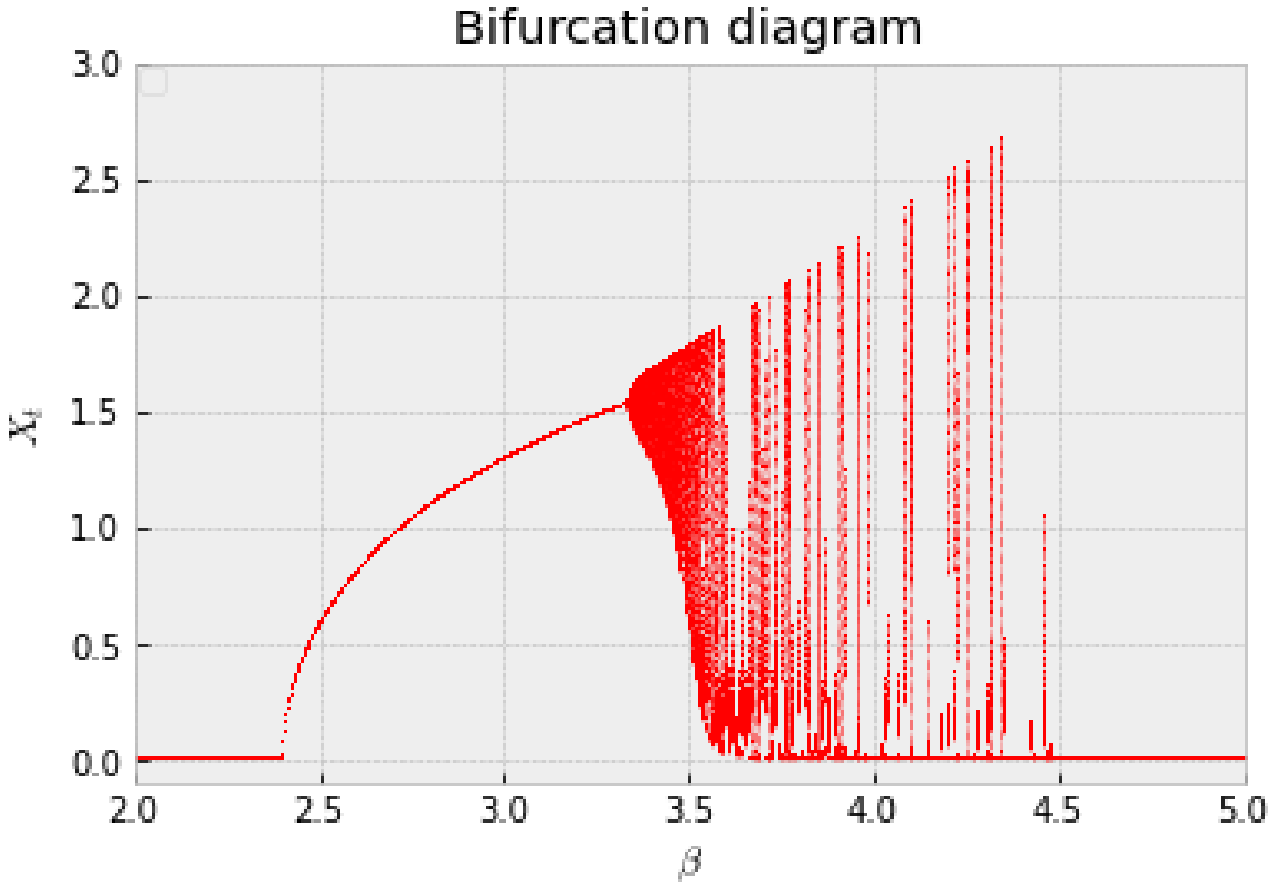
$$R x_t = n_{c,t-1} g x_{t-1} \tag{7}$$

The key feedback loop

- Equation (7) says: $Rx_t = n_{c,t-1} \cdot g \cdot x_{t-1}$
- When chartists dominate ($n_c \to 1$) and $g > R$: the price deviation **amplifies**
- When fundamentalists dominate ($n_f \to 1$): $x_t \to 0$, price returns to fundamental
- The switching mechanism creates an **endogenous feedback**:
  1. Chartists earn more → more agents switch to chartism → price deviates further
  2. Deviation grows → fundamentalists start earning more → switching back
  3. This cycle can produce oscillations, cycles, or chaos depending on β and g
- The profit analysis asks: *who wins this cycle on average?*

Summary

Prices can deviate from fundamentals



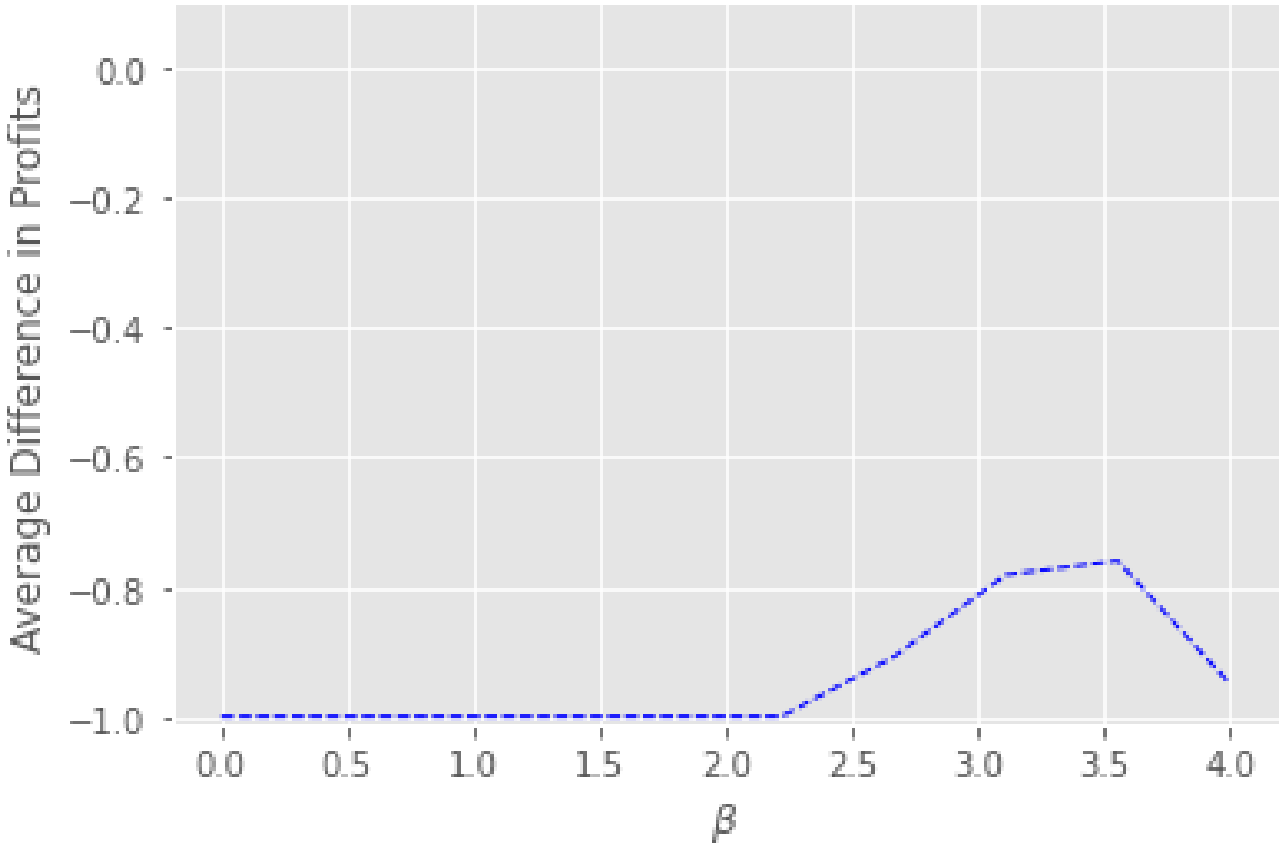
Relative average profits

- We run the model for N (10000) periods
- Calculate the average profits for each of the strategies for the whole period:

$$\hat{\pi}_h = \frac{1}{N} \sum_{t=1}^{N} [\pi_{h,t}] \qquad h = f, c$$

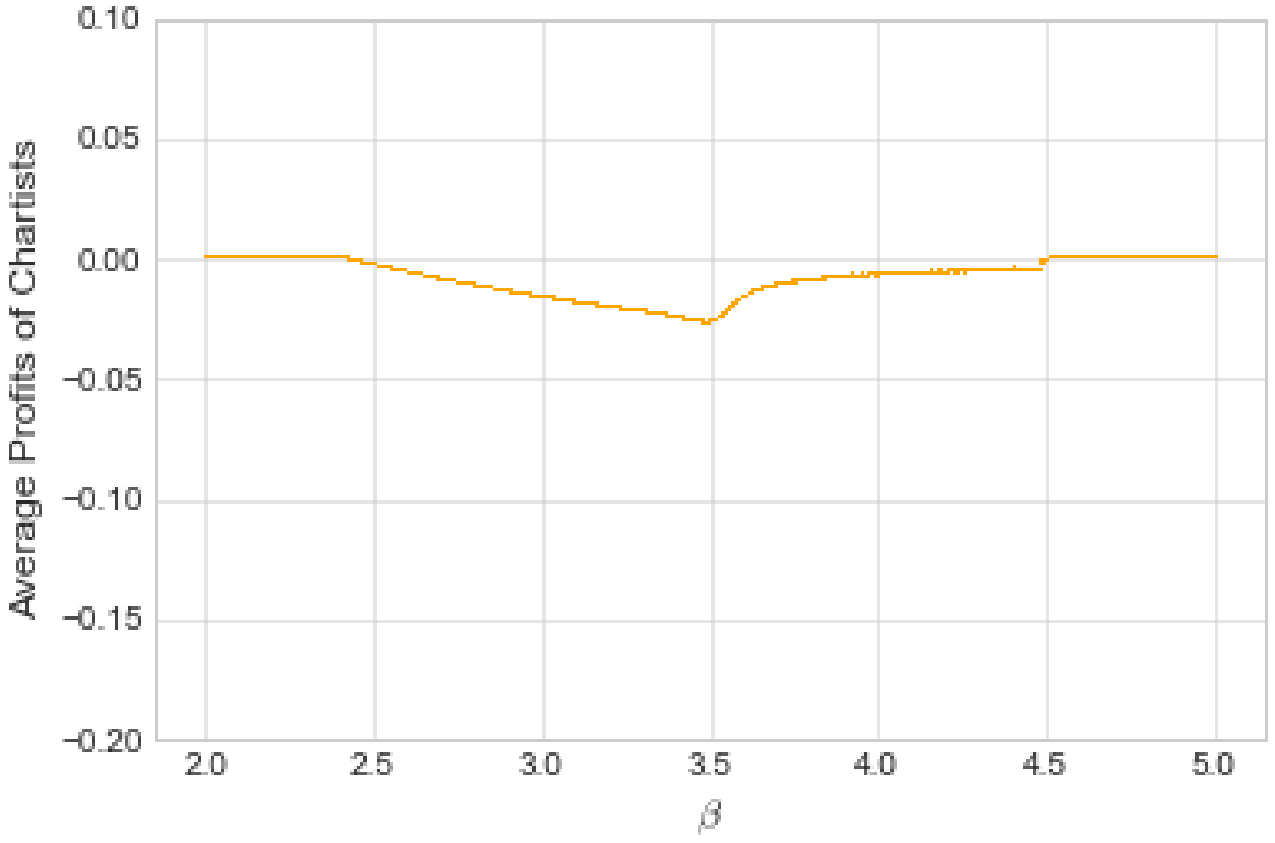
- Calculate $\hat{\pi}_f - \hat{\pi}_c$ for different values of
  - costs C
  - intensity of choice β
- Note: we are comparing **time-averaged** profits, not instantaneous ones
- The key question is whether fundamentalists earn enough *on average* to justify paying C

Fundamentalists gain less



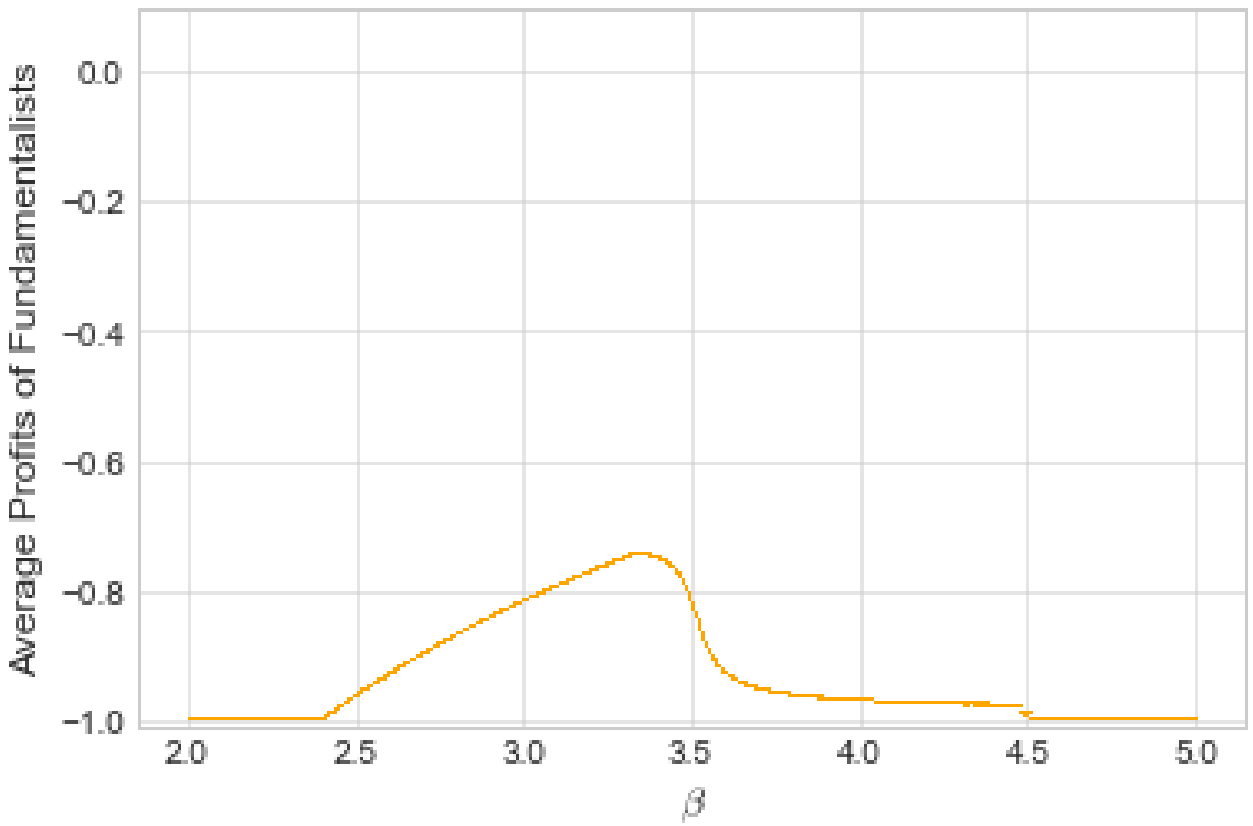
- When price $x_t = 0$, $\hat{\pi}_f - \hat{\pi}_c = -C$ (fundamentalists always pay the cost)
- $\hat{\pi}_f - \hat{\pi}_c$ increases as $x_t > 0$, decreases in part of the chaotic region

Decomposing the difference: chartists



Trend followers also lose on average → not the driver of the difference in profits

Decomposing the difference: fundamentalists



The difference is driven by the profits of fundamentalists

Why do both types lose on average?

- Both $\hat{\pi}_f < 0$ and $\hat{\pi}_c < 0$ for most of the parameter space
- This is not a paradox per se — it reflects the structure of the profit equations
- Recall: $\pi_{h,t} \propto (x_t - Rx_{t-1}) \times (\text{forecast} - Rx_{t-1})$
- When price oscillates, forecasts are **systematically wrong** near turning points
- Both types lose at reversals — but fundamentalists lose *more often* because they are always betting on mean reversion
- The difference $\hat{\pi}_f - \hat{\pi}_c$ is therefore driven by **how much less** fundamentalists lose, not by how much they gain

Profitability paradoxes?

- Is it paradoxical that fundamentalists gain less on average?

Profitability paradoxes?

- Is it paradoxical that fundamentalists gain less on average?
- Not necessarily — agents change strategies over time
  - Fundamentalist strategy is less often profitable with not high profits
  - Fundamentalists bring information to the market and then others can use this

Profitability paradoxes?

- Is it paradoxical that fundamentalists gain less on average?
- Not necessarily — agents change strategies over time
  - Fundamentalist strategy is less often profitable with not high profits
  - Fundamentalists bring information to the market and then others can use this

Paradox 1

As steady state price moves away from its fundamental value, fundamentalists gain relatively more

Profitability paradoxes?

- Is it paradoxical that fundamentalists gain less on average?
- Not necessarily — agents change strategies over time
  - Fundamentalist strategy is less often profitable with not high profits
  - Fundamentalists bring information to the market and then others can use this

Paradox 1

As steady state price moves away from its fundamental value, fundamentalists gain relatively more

Paradox 2

When non-trivial price dynamics emerge, fundamentalists' relative profits are reduced

Paradox 1 — intuition (I)

Paradox 1

As the steady state price moves away from its fundamental value, fundamentalists gain relatively more

- When $x^* > 0$, the price is **persistently above** the fundamental
- Fundamentalists keep forecasting $x = 0$ and are *wrong* — but consistently so
- Their profit formula becomes: $\pi_{f,t} \propto Rx^*(Rx^* - x_t) - C$

Paradox 1 — intuition (II)

Paradox 1

As the steady state price moves away from its fundamental value, fundamentalists gain relatively more

- When x_t oscillates around $x^* > 0$, fundamentalists profit more on the downswing than they lose on the upswing
- Chartists also lose because they extrapolate a trend that keeps reversing
- Net result: larger $x^* \Rightarrow$ larger relative advantage for fundamentalists

Paradox 2 — intuition (I)

Paradox 2

When non-trivial price dynamics emerge, fundamentalists' relative profits are reduced

- In the chaotic region (β large), price dynamics are irregular and unpredictable
- Chartists' trend-following rule occasionally **gets lucky** on chaotic trajectories
- Fundamentalists' mean-reversion forecast performs poorly when there is no stable mean to revert to

Paradox 2 — intuition (II)

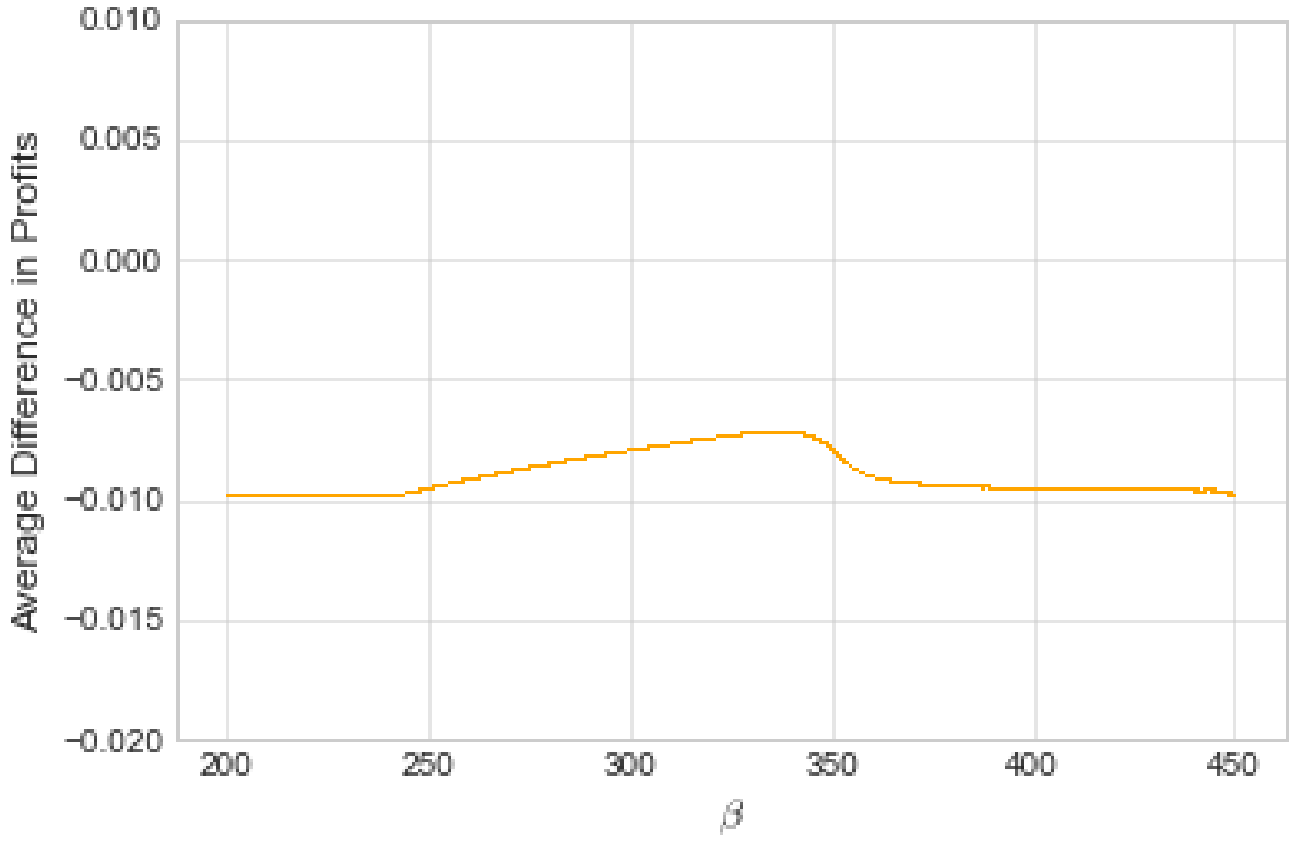
Paradox 2

When non-trivial price dynamics emerge, fundamentalists' relative profits are reduced

- Additionally: in the chaotic region more agents are chartists on average — they attracted followers during the unstable phase — so the denominator of the profit difference shifts
- The bifurcation diagram shows this clearly: the drop in $\hat{\pi}_f - \hat{\pi}_c$ coincides exactly with the onset of complex dynamics around $\beta \approx 3.5$

What if costs are lower?

Average difference in profits for $C = 0.01$



One more paradox?

- Can fundamentalists gain more on average?
- If $C > 0$, then no
- If $C = 0$, only marginally

Paradox 3

Marginal average profits of fundamentalists *decrease* when costs go down

Paradox 3 — intuition (I)

Paradox 3

Marginal average profits of fundamentalists decrease when costs go down

- When C falls, one might expect fundamentalists to be better off — they pay less
- But lower C changes the **equilibrium composition** of the market
- With lower costs, more agents find it attractive to become fundamentalists
- As n_f rises, the price is *pushed closer to the fundamental* more often

This echoes the **Grossman-Stiglitz (1980) paradox**: if prices fully reflect all available information, no one has an incentive to pay to acquire it — but then prices cannot be efficient in the first place

Paradox 3 — intuition (II)

Paradox 3

Marginal average profits of fundamentalists decrease when costs go down

- In Grossman-Stiglitz: when more agents pay for information, prices incorporate it better — reducing the informational advantage and the net return from acquiring it
- In GLR: when more agents become fundamentalists (paying C), prices converge more to the fundamental — reducing deviations x_t and therefore gross profits
- The logic is identical: **the value of a strategy erodes precisely because too many agents adopt it**

Paradox 3 — intuition (III)

Paradox 3

Marginal average profits of fundamentalists decrease when costs go down

- Lower $C \Rightarrow$ more fundamentalists $\Rightarrow$ smaller deviations $\Rightarrow$ smaller gross profits
- The net effect on $\hat{\pi}_f$ is **negative**: the cost saving is more than offset by the reduction in gross earnings
- This is not just a theoretical curiosity — it has a direct policy implication: **making information cheaper does not necessarily make informed trading more profitable**

Conclusion

What we have learned:

1. If you are a fundamentalist it's good to be wrong
2. Driving the price to its fundamental value reduces profits
3. Inconsistency between fundamentalists' beliefs and actual prices

   It's not good to be right for long

Next steps:

- Understand the paradoxes analytically
- Include higher levels of reasoning
  - What if some agents knew the dynamics ex ante?

Part II

Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments

Anufriev & Hommes, *AEJ: Microeconomics*, 2012

Motivation

- BH98 and GLR are **theoretical** models — they assume agents switch based on profits
- But do real people actually behave this way?
- Key question: can the evolutionary switching mechanism explain individual **and** aggregate behaviour in a controlled laboratory setting?
- Anufriev & Hommes (2012) revisit the **learning-to-forecast experiments** of Hommes et al. (2005, 2008)
- They propose a simple **Heuristic Switching Model** (HSM) and show it fits all 20 experimental sessions
- The key result: the same model — with the same parameters — generates three qualitatively different aggregate outcomes depending only on initial conditions

The learning-to-forecast experiment (LtFE)

- 6 human subjects act as advisers to pension funds
- Each period they submit a **price forecast** for a risky asset
- The realized price is determined by market clearing:

$$p_t = \frac{1}{1+r}\left(\bar{p}^e_{t+1} + \bar{y} + \varepsilon_t\right)$$

  where $\bar{p}^e_{t+1}$ is the average of the 6 forecasts
- Subjects are paid based on their forecasting accuracy (quadratic scoring rule)

- Key features:
  - Subjects know r and $\bar{y}$ — they can compute $p^f = \bar{y}/r$
  - They do **not** know others' forecasts or the pricing equation
  - This is a **positive feedback** system: higher forecasts $\Rightarrow$ higher prices

Four experimental treatments

Label	Sessions	Key feature	Outcome
NoRo	HSTV05, 11–14	No robots, range $[0, 100]$	Mixed patterns
LFR	HSTV08, 1–6	No robots, range $[0, 1000]$	Bubbles & crashes
Ro-HF	HSTV05, 1–7	Robots, $p^f = 60$	Dampened oscillations
Ro-LF	HSTV05, 8–10	Robots, $p^f = 40$	Large oscillations

- **Robot traders**: fundamentalists with weight increasing in the price deviation from p^f — they stabilise the market
- **Forecasting range**: wider range removes a psychological anchor and amplifies trend-following behaviour
- The HSM fits all 20 sessions across all four treatments with the **same parameters**

Three different outcomes — same environment

- In sessions with **identical parameters**, three qualitatively different price patterns emerge:
  1. **Monotonic convergence** to $p^f = 60$
  2. **Persistent oscillations** with roughly constant amplitude
  3. **Dampened oscillations** that gradually stabilise
- In the LFR treatment: **long-lasting bubbles** followed by crashes
- Rational expectations is **not** what subjects do

Three different outcomes — same environment

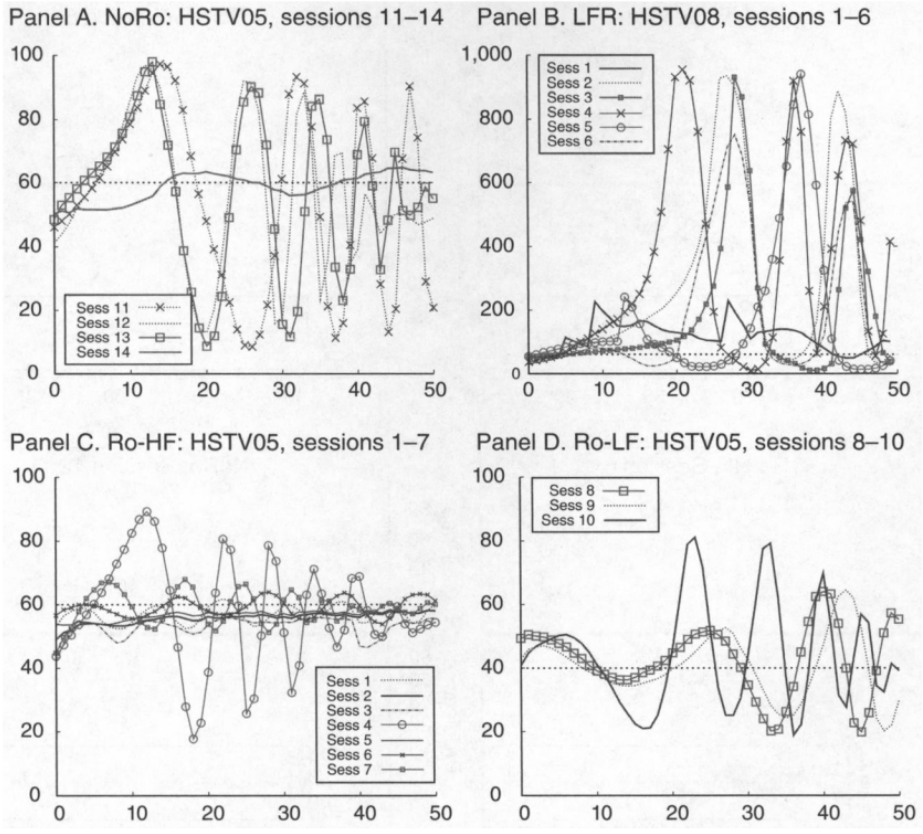


FIGURE 2. AGGREGATE MARKET PRICES IN FOUR DIFFERENT VARIATIONS OF THE LTFE

Aggregate market prices in four different treatments (Anufriev & Hommes 2012, Fig. 2)

Individual forecasting rules

HSTV05 estimated simple linear rules for each subject:

$$p^e_{i,t+1} = \alpha_i + \beta_i p_{t-1} + \gamma_i (p_{t-1} - p_{t-2}) + \delta_i p^e_{i,t}$$

75% of participants are well described by such **first-order heuristics**. Four main types emerge:

Label	**Rule**	**Behaviour**
ADA	$0.65\, p_{t-1} + 0.35\, p^e_t$	Adaptive (stabilising)
WTR	$p_{t-1} + 0.4(p_{t-1} - p_{t-2})$	Weak trend-following
STR	$p_{t-1} + 1.3(p_{t-1} - p_{t-2})$	Strong trend-following
LAA	$0.5(p^{av}_{t-1} + p_{t-1}) + (p_{t-1} - p_{t-2})$	Anchoring & adjustment

- ADA generates convergence; STR generates divergence; LAA generates oscillations

Price dynamics under homogeneous expectations

- What if *all* agents used the same rule? Each heuristic generates a distinct pattern

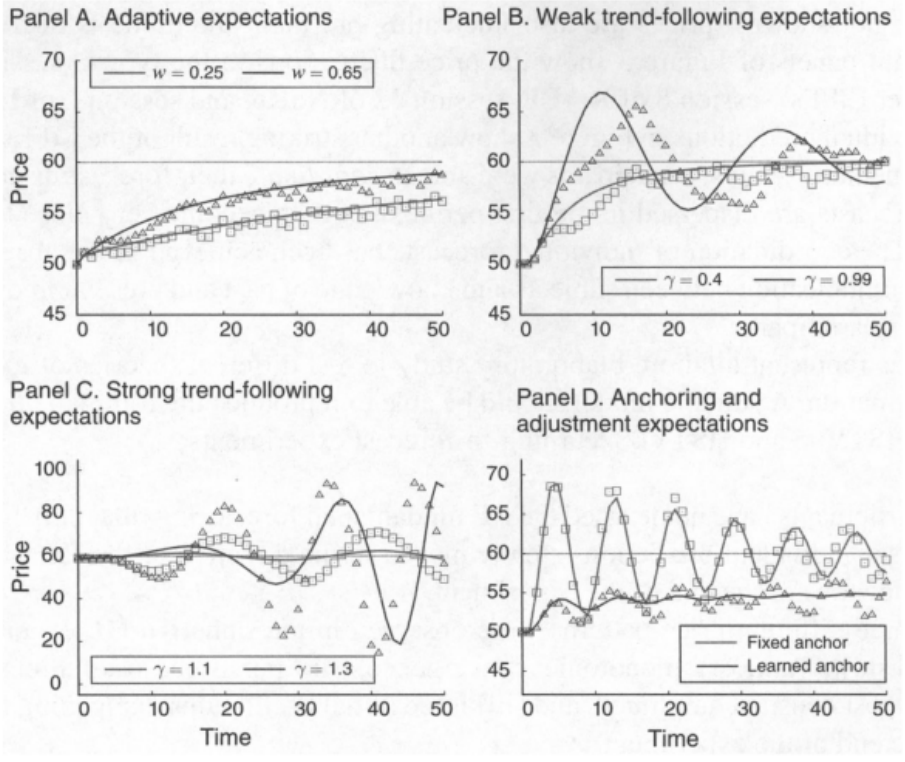
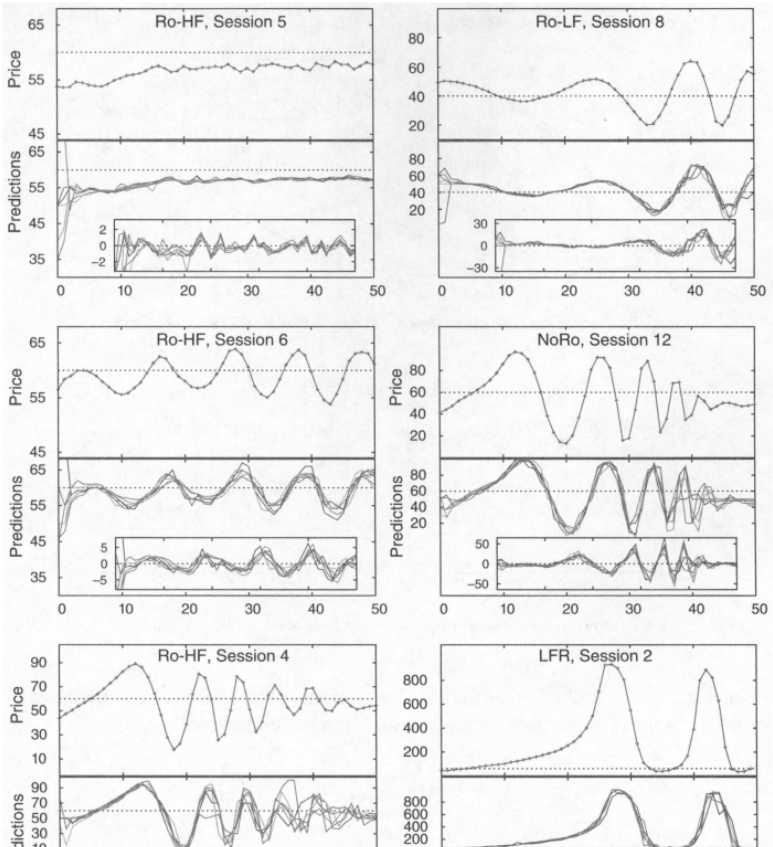


FIGURE 4. PRICE DYNAMICS WITH HOMOGENEOUS EXPECTATIONS

Note: The trajectories of the deterministic skeleton (the curves) and stochastic simulations with the same realization of shocks as in the experiment (triangles and squares) are simulated for different forecasting heuristics.

Coordination and individual switching

- In all sessions, subjects **coordinate** on a common rule — without communicating
- Individual data shows switching between heuristics based on recent performance



Individual switching between rules

- Subjects visibly switch between heuristics over time, responding to past errors

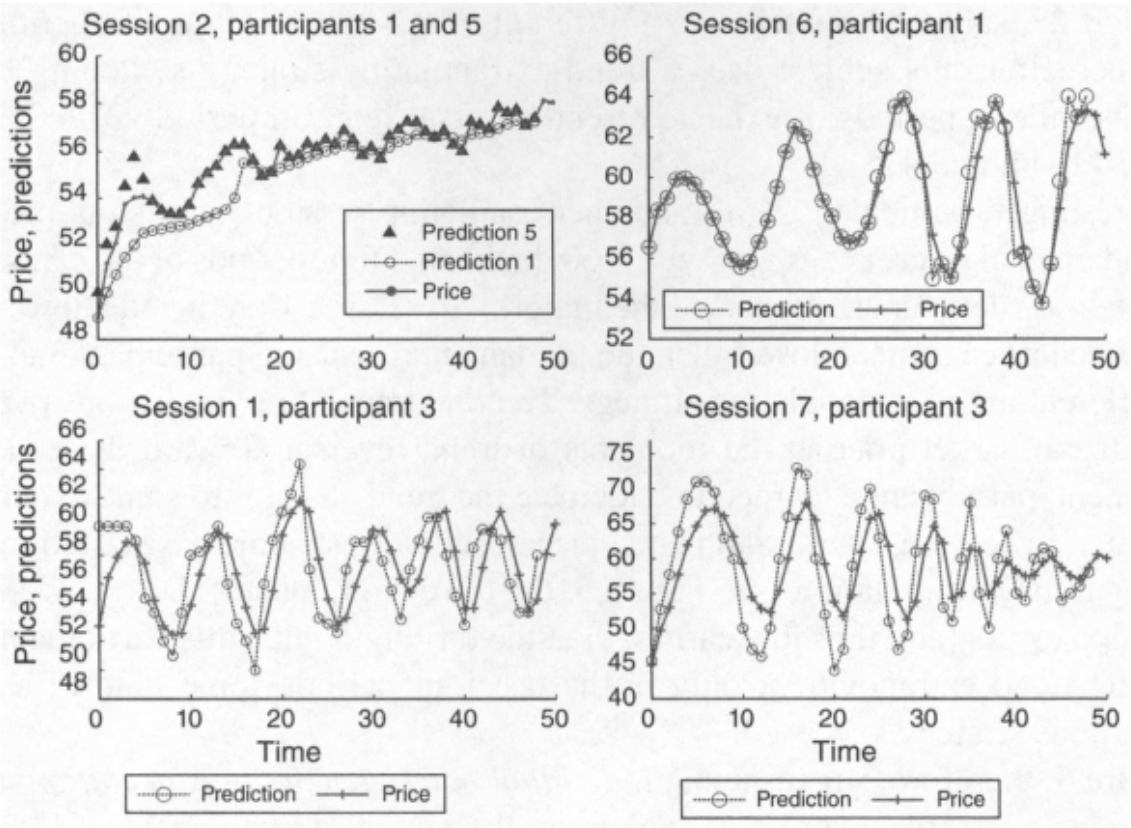


FIGURE 5. SWITCHING OF THE EXPERIMENT'S PARTICIPANTS BETWEEN SIMPLE RULES

Notes: For any point on the abscissa, representing time t, the price p_t and the forecast $p^e_{i,t+2}$ are shown. This forecast, $p^e_{i,t+2}$, was made immediately after the announcement of the price p_t.

The Heuristic Switching Model (HSM)

- The four heuristics compete. The **impact** of rule h evolves over time:

$$n_{h,t} = \delta\, n_{h,t-1} + (1-\delta)\frac{\exp(\beta\, U_{h,t-1})}{Z_{t-1}}$$

  where $\delta \in [0,1]$ is inertia and Z_{t-1} is the normalisation factor

- The **fitness** of rule h is based on past squared forecasting errors (with memory η):

$$U_{h,t-1} = -(p_{t-1} - p^e_{h,t-1})^2 + \eta\, U_{h,t-2}$$

- The aggregate forecast is a weighted average:

$$\bar{p}^e_{t+1} = \sum_{h=1}^{4} n_{h,t}\, p^e_{h,t+1}$$

- Only **three free parameters**: β (intensity of choice), η (memory), δ (inertia)

Path dependence: the key mechanism

- The HSM is a **nonlinear** system — it exhibits path dependence
- If participants initially coordinate on **ADA**:
  - Prices converge ⇒ ADA has small errors ⇒ ADA gains impact ⇒ more convergence
  - **Self-confirming equilibrium**: convergence
- If participants initially coordinate on **STR**:
  - Prices trend ⇒ STR has small errors (initially) ⇒ STR gains impact ⇒ more trending
  - STR misses turning points ⇒ LAA takes over ⇒ persistent oscillations
- The **same parameters** ($\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$) generate all three patterns depending only on initial conditions
- This explains why identical experimental settings produce different outcomes

Empirical fit — benchmark parameters

- With $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$ the HSM fits **all 20 sessions**
- MSE (mean square error) lower than the best homogeneous model in almost every session
- HSM outperforms AR(2) **out of sample**. An **AR(2)** model predicts next period's price using the last two observations:

$$p_t = c + \phi_1 p_{t-1} + \phi_2 p_{t-2} + \varepsilon_t$$

where c is a constant, ϕ_1 and ϕ_2 are estimated coefficients, and ε_t is a white noise term. It is a purely statistical benchmark — no behavioural structure assumed.

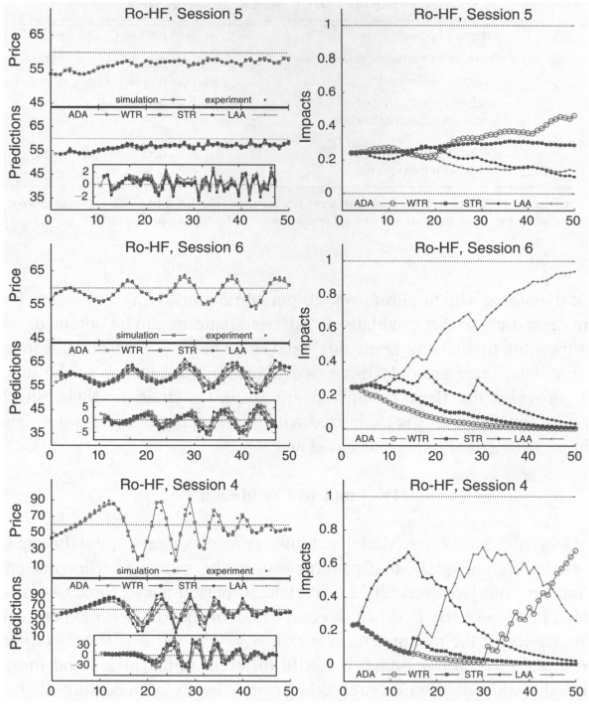


FIGURE 6. LABORATORY EXPERIMENTS AND THE HSM IN THREE SESSIONS FROM **Ro-HF** HSTV05 EXPERIMENT

HSM fit and evolution of heuristic impacts in three Ro-HF sessions (Anufriev & Hommes 2012, Fig. 6)

- **Converging**: ADA dominates **Oscillating**: LAA dominates **Dampened**: STR → LAA → ADA

Connecting back to BH98 and GLR (I)

- BH98: agents switch between fundamentalists and chartists based on past profits
- GLR: who actually earns what along the dynamics?
- Anufriev & Hommes: do real people switch the way the model predicts?

- The HSM is a **direct extension** of the BH97/98 switching mechanism:
  - Same discrete choice structure
  - Fitness based on forecasting performance rather than profits
  - Inertia δ added to reflect experimental evidence of slow updating

Connecting back to BH98 and GLR (II)

Key insight bridging all three papers

Heterogeneity is crucial

- No single homogeneous rule explains all sessions
- The interaction among rules — and their endogenous selection — generates the rich dynamics we observe both in theory and in the lab

- In BH98: heterogeneity in **beliefs** drives non-trivial price dynamics
- In GLR: heterogeneity in **strategies** generates paradoxical profit outcomes
- In AH2012: heterogeneity in **heuristics** explains individual and aggregate behaviour in the lab

Conclusions

From GLR:

1. Fundamentalists earn less on average — even when they stabilise prices
2. They earn relatively more when prices deviate far from the fundamental
3. Lower costs paradoxically reduce their marginal profits
4. **It is not good to be right for long**

From Anufriev & Hommes:

1. Real agents use simple heuristics — not rational expectations
2. They switch between rules based on recent performance
3. Path dependence explains why the same environment produces different outcomes
4. Heterogeneity in expectations is **necessary** to describe individual and aggregate behaviour

Open questions

- Can the paradoxes in GLR be derived analytically — not just numerically?
- What happens if some agents know the dynamics *ex ante*? (higher-order reasoning)
- Can the HSM explain financial market data beyond the laboratory?
  - Allen & Taylor (1990), Frankel & Froot (1990): survey evidence that real traders use trend-following and anchoring rules
- How does the feedback structure (positive vs. negative) affect which heuristics survive?
- Extensions: more than two/four types, stochastic switching, learning the heuristics themselves