Endogenous vs Exogenous Fluctuations: Unveiling the Impact of Heterogeneous Expectations

Macroeconomic Dynamics, 2025

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Where We Come From

Gusella & Ricchiuti (2024, Journal of Evolutionary Economics):

- Heterogeneous Agent Model (chartists & fundamentalists) $\rightarrow$ linear state-space form
- Kalman filter estimation + Monte Carlo simulation
- Evidence of **endogenous damped cycles** in S&P 500
- Rolling-window forecasting: behavioral model vs. random walk

Key limitation:

- Population shares $(\delta, 1-\delta)$ are **fixed**: chartists and fundamentalists always coexist in constant proportions
- Fails to capture **regime switching** — in reality, investors switch strategies over time

$\Downarrow$

This paper: extend to a nonlinear Markov-switching framework

Research Questions

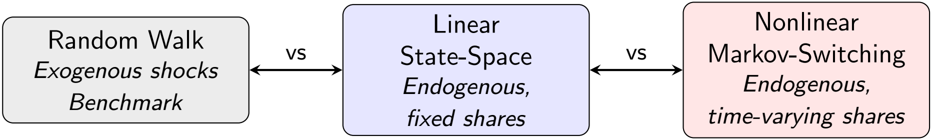
1. Do financial market fluctuations arise from **endogenous** behavioral dynamics or from **exogenous** shocks?

2. Does **nonlinear** (regime-switching) endogenous instability offer greater explanatory power than **linear**, globally stable dynamics?

3. Which specification delivers superior **out-of-sample forecasting** performance?

Data: S&P 500 monthly data, January 1990 – December 2019

Three Competing Models



RW

$p_t = p_{t-1} + \varepsilon_t$

No behavioral content

LSSM

Fixed δ + Kalman filter

Damped cycles

NLMS

Time-varying regime probs.

Local instability

Estimation: Kalman Filter vs. EM Algorithm

Kalman Filter (Path A)

Latent state: **continuous** ($B_t^c \in \mathbb{R}$)

- *Predict:* project B_t^c forward via transition eq.
- *Update:* correct using forecast error on p_t
- Likelihood recovered via prediction error decomposition
- Exact under Gaussianity & linearity

EM Algorithm (Path B)

Latent state: **discrete** ($s_t \in \{f, c\}$)

- *E-step:* compute regime probabilities given current params (Hamilton filter)
- *M-step:* update params maximizing weighted likelihood
- Iterates until convergence
- Guaranteed $\uparrow$ likelihood at each step

Both are **frequentist** — no prior, no posterior.

Difference from Lux (2018, 2021): Bayesian particle/MCMC filters.

Monte Carlo Simulation: Why and How

Why? MLE on small-to-medium samples is sensitive to local optima and starting values — a single optimization run may not find the global maximum.

How?

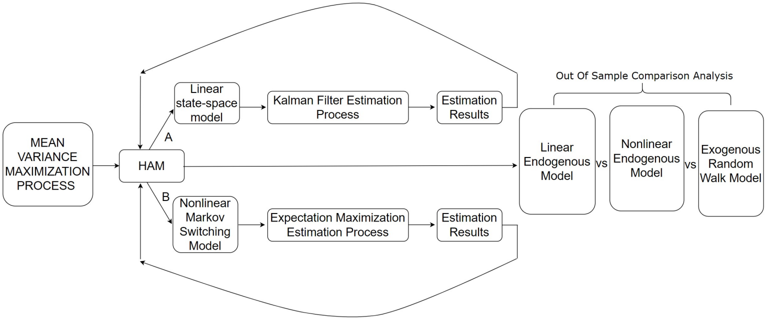
- Draw **1000 random starting points** for the parameters
- Run Kalman (or EM) estimation from each starting point
- Retain the solution with the **highest likelihood**
- Report mean and confidence intervals across replications

What it is not:

- Not a Bayesian procedure — the 1000 draws are starting points, not posterior samples
- Not a bootstrap — we simulate from the model, not from the data

Robustness check against local optima, not a source of uncertainty quantification.

Methodology



Path A: HAM → linear state-space → Kalman filter estimation → out-of-sample comparison.

Path B: HAM → nonlinear Markov-switching → EM estimation → out-of-sample comparison.

The HAM Setup

Standard mean-variance optimizing agents; two types:

Asset pricing equation:

$$p_t = \delta\, E_t^f(p_{t+1}) + (1-\delta)\, E_t^c(p_{t+1})$$

Fundamentalists (anchor to Gordon-model fundamental p_t^f):

$$E_t^f(p_{t+1}) = p_t^f$$

Chartists (extrapolate past price changes, first-order heuristic):

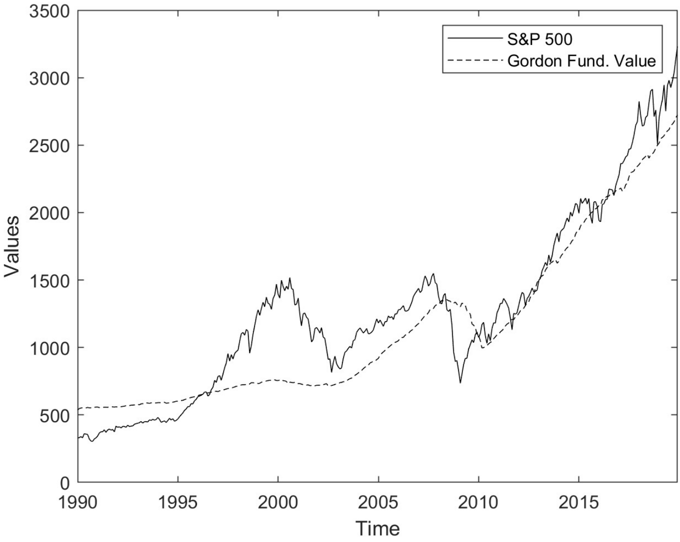
$$E_t^c(p_{t+1}) = p_t^f + \beta(p_{t-1} - p_{t-2}), \quad \beta > 0$$

Substituting:

$$p_t = p_t^f + \underbrace{(1-\delta)\,\beta(p_{t-1} - p_{t-2})}_{B_t^c}$$

⇒ Price deviates from fundamental through the **unobserved belief** B_t^c

Data: S&P 500 and Fundamental Value



Path A: Linear State-Space Form

Substituting into the belief function of chartists and using the random-walk fundamental:

$$B_t^c = \beta(1-\delta)\,B_{t-1}^c - \beta(1-\delta)\,B_{t-2}^c + \varepsilon_t$$

Measurement equation:

$$p_t = p_t^f + \begin{bmatrix}(1-\delta) & 0\end{bmatrix}\begin{bmatrix}B_t^c \\ B_{t-1}^c\end{bmatrix}$$

Transition equation:

$$\begin{bmatrix}B_t^c \\ B_{t-1}^c\end{bmatrix} = \underbrace{\begin{pmatrix}a_{11} & a_{12} \\ 1 & 0\end{pmatrix}}_{A}\begin{bmatrix}B_{t-1}^c \\ B_{t-2}^c\end{bmatrix} + \begin{bmatrix}\varepsilon_t \\ 0\end{bmatrix}$$

with $a_{11} = \beta(1-\delta)$, $a_{12} = -\beta(1-\delta)$.

Cyclical condition (complex eigenvalues of A):

$$0 < \beta(1-\delta) < 4$$

If $\sqrt{\beta(1-\delta)} < 1$: *damped* endogenous fluctuations.

Estimation: Kalman filter + Monte Carlo MLE (1000 repetitions)

Path B: Nonlinear Markov-Switching Form

Agent shares δ and $(1-\delta)$ are **time-varying**, governed by a first-order Markov chain $s_t \in \{f, c\}$.

Working in first differences:

$$\Delta p_t = \varepsilon_t(s_t) + \left[a_{11}\,\Delta p_{t-1} + a_{12}\,\Delta p_{t-2}\right](s_t)$$

State transition matrix:

$$\begin{pmatrix} p_{ff} & p_{fc} \\ p_{cf} & p_{cc} \end{pmatrix}, \quad p_{ff} + p_{fc} = 1, \quad p_{cf} + p_{cc} = 1$$

Regime f (fundamentalist):
price tracks p_t^f

Regime c (chartist):
price driven by past values

The **filtered regime probability** serves as a time-varying proxy for the weight assigned to each behavioral strategy.

Estimation: Expectation-Maximization (EM) algorithm + Monte Carlo (1000 repetitions)

In-Sample Results: Linear Model (LSSM)

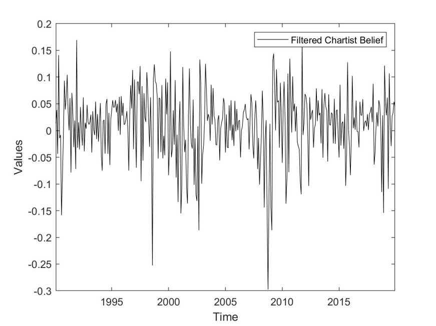
	Cyclical Parameters	
	a_{11}	a_{12}
Values	0.5398^{***} [0.537, 0.542]	-0.5398^{***} [−0.542, −0.537]

	δ	$1-\delta$	β
Values	0.69	0.31^{***}	1.74

Cycl. cond.	**Fulfilled**
Eigenvalues	$-0.270 \pm 0.683\,i$
Modulus	0.73 (*damped*)

- 69% fundamentalists, 31% chartists
- **Damped cycles**: modulus $0.73 < 1$
- $B_t^c > 0$ in booms; sharp $\downarrow$ at GFC

Filtered chartist belief B_t^c (Fig. 3)



In-Sample Results: Nonlinear Model (NLMS)

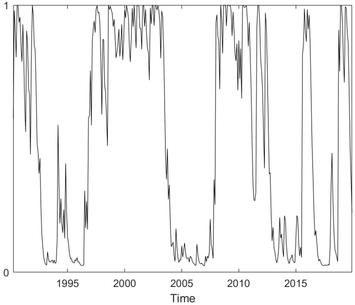
	Cyclical Parameters	
	a_{11}	a_{12}
Values	0.04^{***}	-0.04^{***}
	[0.03, 0.05]	[−0.05, −0.03]

Transition probabilities	
$p_{ff} = 0.9425$	$p_{fc} = 0.0575$
$p_{cf} = 0.0833$	$p_{cc} = 0.9167$

Cycl. cond.	**Fulfilled**
Eigenvalues	$-0.020 \pm 0.199\,i$
Modulus	0.46 (*damped*)

- High persistence: agents stay in same regime
- Chartist regime ↑ at dot-com & GFC
- β time-varying: spikes & reverts

Chartist regime prob (Fig. 4)



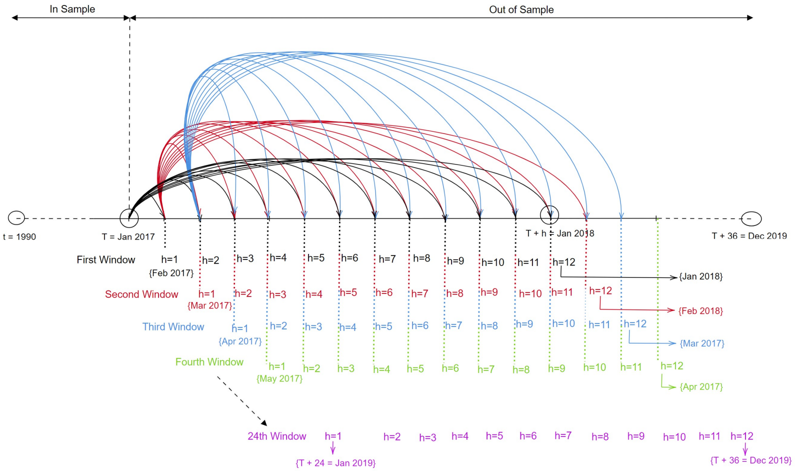
Rolling-Window Out-of-Sample Design

- **In-sample:** Jan 1990 – Dec 2016 **Out-of-sample:** Jan 2017 – Dec 2019
- **Rolling window:** 325 monthly observations; window shifts by 1 each step
- **Forecast horizons:** $h = 1, 2, \ldots, 12$ months
  - Short-run: $h = 1, 2, 3, 4$
  - Medium-run: $h = 5, 6, 7, 8$
  - Long-run: $h = 9, 10, 11, 12$
- Forecast accuracy: **RMSE** and **MAE** ratios + **Diebold-Mariano** test

Ratios reported:

- Table 4: RMSE(LSSM) / RMSE(RW) > 1 means RW wins
- Table 5: RMSE(NLMS) / RMSE(LSSM) < 1 means NLMS wins
- Table 6: RMSE(NLMS) / RMSE(RW) < 1 means NLMS wins

Rolling-Window Out-of-Sample Design



At each step, the model is re-estimated on the most recent 325 observations and forecasts are generated for $h = 1, \ldots, 12$ months ahead.

Comparing Forecast Accuracy: Diebold-Mariano Test

Question: are the differences in forecast accuracy statistically significant, or just due to sampling variability?

Define the **loss differential** between two forecasts (e.g. NLMS vs. LSSM):

$$d_{T+h} = \left(\varepsilon^{L}_{T+h|T}\right)^2 - \left(\varepsilon^{NL}_{T+h|T}\right)^2$$

Null hypothesis: equal predictive accuracy

$$H_0 : \mathbb{E}[d_{T+h}] = 0 \quad \forall\,(T+h)$$

DM test statistic:

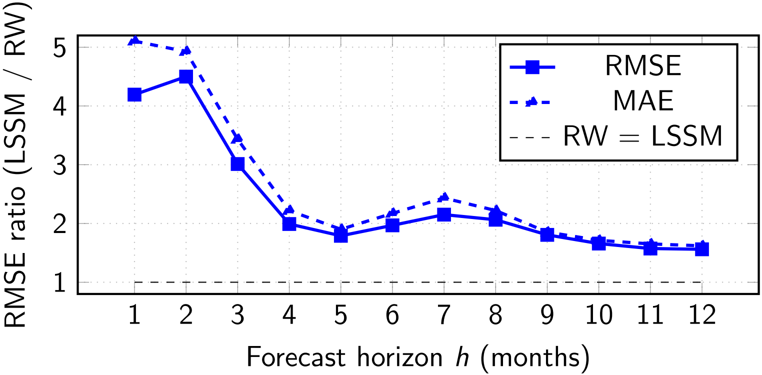
$$DM = \frac{\bar{d}}{\sqrt{\widehat{\sigma}^2_d / k_h}} \xrightarrow{d} \mathcal{N}(0,1)$$

where $\bar{d}$ is the sample mean of d_{T+h}, $\widehat{\sigma}^2_d$ its long-run variance, and $k_h = 24$ the number of forecast observations.

- Applied to all three pairwise comparisons (Tables 4, 5, 6)
- Rejection of $H_0 \Rightarrow$ the performance gap is **statistically significant**
- Results robust at 1% level for NLMS vs. LSSM at all horizons

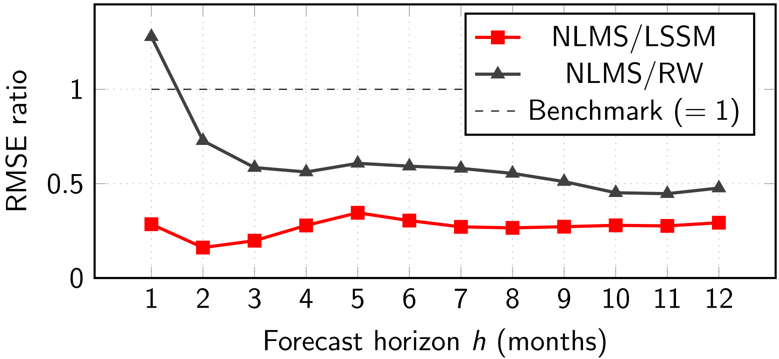
Out-of-Sample: LSSM vs. Random Walk (RMSE ratio)

Ratio > 1: RW outperforms LSSM at all horizons; gap narrows as h grows.



Interpretation: LSSM converges toward RW at long horizons — consistent with its mean-reverting structure.

Out-of-Sample: NLMS vs. LSSM and RW (RMSE ratio)



- **NLMS vs LSSM:** ratio ≈ 0.15–0.35 at *all* h (1% sig.)
- **NLMS vs RW:** beats RW from $h = 2$; ratio $\rightarrow 0.45$ at $h = 12$

Summary of Out-of-Sample Results

Model	Short-run	Medium-run	Long-run
LSSM vs RW	RW wins	RW wins	RW wins*
NLMS vs LSSM	NLMS wins	NLMS wins	NLMS wins
NLMS vs RW	$\approx$ tied	NLMS wins	NLMS wins

* LSSM converges toward RW performance at long horizons

Bottom line: nonlinear endogenous dynamics $\succ$ linear endogenous dynamics $\succ$ exogenous RW (at medium/long horizons)

Comparison with Gusella & Ricchiuti (2024)

	GR (2024)	**DGR (2025)**
Agent shares	Fixed (δ constant)	Time-varying (Markov chain)
Estimation	Kalman filter (MLE)	EM algorithm
Models compared	LSSM vs. RW	NLMS vs. LSSM vs. RW
Reaction coeff. β	Constant	Time-varying
Regime detection	No	Yes (filtered probs.)
Out-of-sample	LSSM beats RW (medium/long)	NLMS beats both (from $h = 2$)

Novelty of DGR (2025):

- First paper to compare *linear* vs. *nonlinear* endogenous models against the RW in a state-space out-of-sample forecasting context
- Markov-switching captures **local instability** episodes (dot-com, GFC)

Conclusions

1. **Endogenous fluctuations confirmed:** both LSSM and NLMS exhibit complex eigenvalues, supporting behavioral heterogeneity as the driver of financial instability.
2. **Nonlinearity matters for forecasting:** the Markov-switching model substantially outperforms the linear behavioral model at all horizons.
3. **Beating the random walk:** NLMS outperforms the RW from $h = 2$ onward; advantage grows with the horizon, reaching ∼50% RMSE reduction at $h = 12$.
4. **Regime interpretation:** trend-following dominates during market stress; fundamentalist behavior guides recovery.

Future extensions: cripto currencies, additional heuristics (noise traders, contrarians); exchange rate market; multivariate models linking financial and real sectors.